

DETECTION OF SEISMIC DAMAGE IN BUILDINGS USING STRUCTURAL RESPONSES

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NON-TECHNICAL ABSTRACT

As a result of an earthquake and its aftershocks built infrastructure may sustain damage. One of the major challenges for quick and efficient recovery in the aftermath of a hazardous event is rapid estimation of the damage. If the state of buildings, bridges, dams and other structures could be quickly and reliably assessed healthy, undamaged structures could be immediately re-opened for continuous, uninterrupted service, while damaged structures would be closed and prioritised for later detailed evaluation, repair, demolition or replacement. Doing so will minimize casualties and economic losses and will aid quick recovery of an affected area. Accurate estimation of seismic damage is, however, a time and resource consuming task. Traditionally, it can be achieved by visual inspection of infrastructure following an earthquake. However, given the usually large stocks of structures to inspect and limited number of qualified personnel damage assessment is a slow process. The fact that damage can often be inconspicuous adds to the difficulty.

An alternative to visual inspection can be using measurements of structural responses during strong motion events taken by sensors located in the structure. This approach becomes feasible with the development of continuous seismic monitoring arrays. In New Zealand, the EQC and FRST funded GeoNet monitoring project that is currently expanding its coverage to buildings and bridges, can be used for structural damage detection. However, raw data from seismic sensors are of limited value. The challenge is to analyse the measured structural responses so that structural damage can be detected and quantified. This research studies several techniques that enable such purposeful data analyses.

Damage detection by analysis of structural responses is based on the premise that it is possible to choose certain response signal features that are different for responses of healthy and damaged structures. Once the features are selected another analytical tool is required to actually tell the difference between the features corresponding to different structural states. In this research, we modelled structural accelerations using autoregressive time series models in order to find suitable damage sensitive features, and used pattern recognition techniques for feature classification. The approach was thoroughly investigated through several experimental studies and results of damage detection and quantification are promising.

TECHNICAL ABSTRACT

The ability to estimate seismic induced damage to civil infrastructure is undoubtedly one of the most important challenges faced by structural engineers. In this research, structural damage was detected and assessed by analysing the structural response.

Autoregressive (AR) time series models were used to fit the acceleration time histories obtained when the structure was in both undamaged and damaged states. The AR coefficients were selected as damage sensitive features and statistical pattern recognition techniques were investigated for interpreting changes in the values of these features caused by damage. Initially, an offline damage detection method was developed in which Back-Propagation Artificial Neural Networks (BP ANNs) were used for both classification and quantification tasks where the damage states were recognized or percentage of remaining stiffness at a specific location was estimated, respectively. The method was applied to three experimental structures: a 3-storey bookshelf structure, the ASCE SHM Phase II Experimental Benchmark Structure and a RC column. In addition, for damage classification tasks, two supervised classification techniques of Nearest Neighbour Classification (NNC) and Learning Vector Quantisation (LVQ), and an unsupervised method of Self-Organising Maps were studied. Damage classification and/or quantification using BP ANNs, NNC and LVQ techniques was achieved with very good results confirming the usefulness of AR coefficients as damage sensitive feature and the studied pattern recognition techniques as damage classifiers.

An online damage detection method was also developed based on recursive identification of the AR models using the forgetting factor and Kalman filter approaches and BP ANNs. An analytical linear 3-DOF model with time varying stiffness was investigated and the results showed that damage could be detected and quantified as it occurred. Damage detection in nonlinear systems was addressed with the investigation of an analytical 1-DOF elastoplastic oscillator and a 3-DOF Bouc-Wen hysteretic model. In both cases the on-set of nonlinearity was detected with good accuracy.

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LIST OF ABBREVIATIONS

ANN	Artificial Neural Network
AR	Autoregressive
ARMAX	Autoregressive-Moving Average with eXogenous input
ARX	Autoregressive with eXogenous input
ASCE	American Society of Civil Engineers
BP	Back-Propagation
CSD	Cross-Spectral Density
DFT	Discrete Fourier Transform
DOF	Degree Of Freedom
ERA	Eigenvalue Realisation Algorithm
FEM	Finite Element Method
FRF	Frequency Response Function
IFR	Impulse Response Function
LVQ	Learning Vector Quantisation
MA	Moving Average
MAC	Modal Assurance Criterion
MDOF	Multiple Degree Of Freedom
NNC	Nearest Neighbour Classification
PCA	Principal Component Analysis
PEM	Prediction Error Method
PGA	Peak Ground Acceleration
PSD	Power Spectral Density
RC	Reinforced Concrete
SDOF	Single Degree Of Freedom
SHM	Structural Health Monitoring
SI	Spectrum Intensity
SOM	Self-Organising Maps

NOTATION

The following notation is used throughout this report:

$\mathbf{0}$	null matrix
$\mathbf{A}$	state matrix
a	AR coefficient
$\mathbf{B}$	input matrix
B	backshift operator
b	exogenous coefficient
$\mathbf{C}$	damping matrix, output matrix
c	MA coefficient, damping coefficient
$\mathbf{D}$	feedthrough matrix
D	distance between vectors
$\mathbf{d}$	vector of desired ANN outputs
$\mathbf{E}$	selection matrix
E	error, expectation operator
$\mathbf{e}$	error vector
e	error
F	discrete Fourier transform of input, frequency
f	frequency
$\mathbf{H}$	Hankel matrix, covariance matrix of noise
H	frequency response function
$\mathbf{I}$	identity matrix
I	imaginary unit
Im	imaginary part
$\mathbf{J}$	Jacobian matrix
$\mathbf{K}$	stiffness matrix
k	stiffness
$\mathbf{L}$	gain matrix
$\mathbf{M}$	mass matrix
$\mathbf{m}$	codebook vector
m	mass
na	AR order

nb	exogenous order
nc	MA order
$\mathbf{o}$	vector of ANN outputs
$\mathbf{P}$	covariance matrix
$\mathbf{Q}$	covariance matrix of noise
$\mathbf{R}$	matrix of singular vectors
R	autocorrelation function, cross-correlation function
Re	real part
$\mathbf{S}$	matrix of singular vectors, sensitivity matrix
S	auto-spectral density, cross-spectral density
$\mathbf{T}$	transition matrix
T	natural period
t	time
$\mathbf{u}$	state-space input vector
u	weighted sum of inputs, displacement, input
$\mathbf{V}$	matrix of singular vectors
$\mathbf{w}$	ANN weights vector
X	discrete Fourier transform of response
$\mathbf{x}$	state vector, feature vector
x	input, excitation
$\mathbf{Y}$	matrix of previous time series output
$\mathbf{y}$	state-space output vector, vector of current time series outputs
y	time series, output
$\mathbf{Z}$	measurement matrix
z	principal component
Δ	interval, increment
η	learning rate
θ	vector of time series model coefficients, vector of updating parameters
Λ	matrix of singular values
λ	iteration parameter, eigenvalue, forgetting factor
ξ	damping ratio
Σ	covariance matrix, matrix of singular values

σ	standard deviation
Φ	mode shape
ϕ	mode shape
φ	vector of previous time series values
ω	radial frequency

$\{\}$	time series
$ $	absolute value

Subscripts:

AR	AR coefficients
a	analytical
$accel$	accelerometers
c	continuous time system, closest codebook vector, complex number
$crit$	critical
E	Euclidean metric
e	experimental
i	index, iteration step
j	index
f	input
k	time step, index, iteration step
M	Mahalanobis metric
m	measurements
r	real part, input
s	sampling frequency
x	output

Superscripts:

T	matrix transpose
$\wedge$	estimated quantity
$-$	mean value
$+$	pseudoinverse

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CHAPTER 1

INTRODUCTION

1.1. Background and motivation for research

In seismically active regions, such as New Zealand, the ability to detect and quantify seismic induced damage to civil infrastructure is undoubtedly one of the most important challenges faced by structural engineers. In the aftermath of a major earthquake, an efficient and reliable method for assessing the extent of damage to civil infrastructure would enable the immediate reoccupation of structures identified as having sustaining only minor damage and informed decisions to be made on the repair or demolition of structures identified as sustaining major damage. This research develops methods for damage detection and quantification from analysis of vibration data recorded on structures under dynamic excitations.

Various structural monitoring programmes have already begun abroad and are now being implemented in New Zealand with the GeoNet programme (Gledhill et al. 2006). Subsequent analysis of the vibration data gathered by monitoring systems could reveal changes in the structural response and identify the presence of damage. Such studies are commonly referred to as Structural Health Monitoring (SHM) which can be broadly defined as a process involving firstly, tracking any aspect of structural performance or health by measuring data and secondly, interpreting changes in these data so that structural condition and reliability can be quantified objectively (Aktan et al. 2002). During the past decade research into SHM has received considerable attention. This can be partly attributed to the recent advances in electronic, sensor and wireless technologies together with the development of analytical tools. Despite this interest, a robust and reliable SHM system capable of detecting, locating, and quantifying damage while remaining unaffected by changes in environmental and operating conditions has yet to be agreed upon.

SHM methods can be broadly divided into several paradigms. Amongst these, vibration based methods appear to be the most promising. These methods use damage sensitive features extracted from the structure's dynamic response to identify damage. Various techniques have been proposed either in the frequency domain (Wu et al. 1992; Zang and Imregun 2001; Ni et al. 2006) or time domain (Nakamura et al. 1998; Nichols et al. 2003; Yang et al. 2007). Motivated by the relatively unexplored, although promising use of time series analysis techniques in SHM, this research attempts to develop a health monitoring methodology using these techniques.

1.2. Objective, contribution and scope of research

The objective of this research is to apply the tools of statistical pattern recognition, in particular Artificial Neural Networks (ANNs), to develop methods of damage detection and quantification for civil infrastructure that may be affected by seismic induced damage. The contribution and scope of the research is outlined below.

Methods for damage detection and assessment using Autoregressive (AR) time series models and statistical pattern recognition techniques were investigated. AR models were used to fit the acceleration response of structures with varying degrees of damage. The AR coefficients were selected as damage sensitive features and statistical pattern recognition techniques were applied to interpret changes in the values of these coefficients caused by damage. The contribution of this research was the development of an offline method for detecting, locating and quantifying damage in civil infrastructure using AR coefficients and BP ANNs. The AR coefficients were used as inputs into the BP ANN and the ANN was trained to recognise changes in the patterns of coefficients and relate these to either a specific damage state (damage classification) or a reduction in stiffness (damage quantification). This methodology allowed for both the location of damage to be identified and extent of damage to be quantified. The method was applied to three experimental structures: a 3-storey bookshelf structure, the ASCE Phase II SHM Experimental Benchmark Structure and a RC column. The results showed the combination of AR models and ANNs were effective at damage classification and quantification tasks. The use of data reduction techniques was investigated for systems in which the number of sensors and/or AR coefficients proved to be too large for practical application of BP ANNs. Principal Component Analysis (PCA) was used to reduce the dimensionality of the feature and allowed BP ANNs to continue to be practical for these systems.

In addition, research contributions were made with novel application of Nearest Neighbour Classification (NNC), Learning Vector Quantisation (LVQ) and Self-Organising Maps (SOM) techniques in the civil infrastructure field. The basic idea behind these methods is to establish a database of baseline damage features corresponding to various damage states, and later, when a new feature becomes available, assign it to the damage state with the closest distance between the new feature and the reference feature clusters. For damage classification tasks, NNC and LVQ supervised learning methods were found to be effective damage classifiers. SOM, an unsupervised method, showed promising results. Visualisation of damage data was attempted using 2D projections of the AR coefficients obtained using PCA and Sammon mapping. For the ASCE Phase II SHM Experimental Benchmark Structure these projections showed distinct clusters corresponding to various damage states.

Previously outlined methods used AR coefficients estimated using an offline identification technique allowing for an intermittent damage prognosis. Another contribution of this research was the development of an online method for detecting and assessing damage in real-time using recursive identification of AR coefficients and BP ANNs. The forgetting factor and Kalman filter approaches were investigated for the identification of AR coefficients. Applied to an analytical model of a 3-DOF linear elastic oscillator with time-dependent stiffness, the results showed that damage could effectively be tracked as it accumulated. The concept of damage detection in nonlinear systems was investigated on analytical models of a 1-DOF elastoplastic oscillator and a 3-DOF Bouc-Wen hysteretic system. Damage could no longer be regarded as a simple reduction in stiffness and instead the presence of nonlinearity was successfully detected by observing sudden changes in the values of AR coefficients.

1.3. Report layout

The layout of this report is as follows:

Chapter 2 – A review of contemporary approaches to damage detection is given. A brief outline of the historical development of ANNs and applications to engineering and specifically structural engineering is also presented.

Chapter 3 – A description of the three experimental structures used in this study: a 3-storey bookshelf structure, the ASCE Phase II SHM Experimental Benchmark Structure, and a RC column are given and the results of dynamic testing on these structures are presented.

Chapter 4 - An offline damage detection method using AR models and BP ANNs is developed and experimentally verified on the three experimental structures.

Chapter 5 - Three statistical pattern recognition techniques: NNC, LVQ and SOM are studied and compared against BP ANNs for damage classifications tasks. Reduction and visualisation of damage data is investigated using PCA and Sammon mapping.

Chapter 6 - An online damage detection method is developed which uses recursive identification of the AR models. Online damage detection in linear and nonlinear systems is investigated.

Chapter 7 – Conclusions and recommendations for further research are provided.

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CHAPTER 2

LITERATURE REVIEW

This literature review consists of two parts. Firstly, an introduction to Artificial Neural Networks (ANNs), the main tool used for damage detection and quantification in this research, with key historical developments and applications is given. Secondly, current trends in Structural Health Monitoring (SHM) methods are reviewed emphasising civil infrastructure applications.

2.1. Artificial Neural Networks

ANNs are data processing structures deliberately designed to utilise the organisational principles found in the brain (Anderson and Rosenfeld 1988). Consisting of self-organising, interconnected layers of simple computational units or neurons, their parallel architecture allows for a powerful information processing system capable of classification, pattern recognition and functional mapping tasks. Over 60 years of research has lead to several different types of neural networks including the Boltzmann machine (Hinton and Sejnowski 1986), Back-Propagation (BP) (Rumelhart et al. 1986), competitive learning (Rumelhart and Zipser 1986), Central-Propagation (CP) (Hecht-Nielsen 1987), Hopfield (Hopfield 1982), Kohonen (Kohonen 1984), Learning Vector Quantization (LVQ) (Kohonen 1988), and Radial Basis Function (RBF) (Poggio and Girosi 1989) neural networks. These networks vary in topology and method of learning, which can be supervised or unsupervised. Rumelhart and McClelland (1986) gives a comprehensive overview of the basic anatomy of ANNs. A more recent review of the major types of ANNs, with an emphasis on computation, can be found in Freeman and Skapura (1992).

2.1.1. Historical development

Early research into mathematical models of brain function was for the primary purpose of scientific discovery into areas of neurobiology, cognition, psychology and human behaviour. A compilation of major discoveries in neural networks can be found in Anderson and Rosenfeld (1988). A collection of these founding articles in the development of artificial neural networks is presented below.

The origins of ANNs stem from a paper written by McCulloch and Pitts (1943) in which they theorised a computational model of a neuron. They considered the neuron to be a binary device activated when the input exceeded a certain threshold. Arranging these neurons into nets with the correct thresholds allowed simple logic operations to be made. Although McCulloch and Pitts did not explicitly state that the computational power of these simple neurons was due to their interconnectivity, it was obvious to them (Anderson and Rosenfeld 1988). McCulloch and Pitts concluded that the brain was a large logic computational device.

The first physiological learning rule was theorised by Hebb (1949) who introduced the idea of synaptic modification, a method of learning by adjustment of neuron weights. Rochester et al. (1956) simulated this learning rule and the accompanying theory of Hebb on a computer. Further developments were made by Rosenblatt (1958) when the first computationally oriented neural network, the perceptron (Anderson and Rosenfeld 1988) was introduced. The perceptron architecture consisted of a connected sensory, association and response layer. Widrow and Hoff (1960) introduced the adaptive neuron and the least mean squares algorithm for supervised learning. They demonstrated the results on the purpose built ADALINE (ADaptive LInear NEuron) machine, an adaptive pattern classification machine. Minsky and Papert (1969) placed computational limits on the perceptron with a detailed mathematical analysis. This excellent piece of research almost stopped the development of neural networks entirely (Anderson and Rosenfeld 1988; Freeman and Skapura 1992).

Over two decades passed before neural networks made a full recovery from the findings of Minsky and Papert (1969). During this intermediary period only a small amount of research continued into neural networks and improvements were made on the perceptron with the simultaneous discoveries of Kohonen (1972) and Anderson (1972). Both of these studies presented a model of associative memory, a structure that mapped a set of input patterns to a set of output patterns. The model is now referred to as the linear associator.

Kohonen (1982) introduced the Self-Organising Map (SOM), an unsupervised learning technique in which neurons through a process of self-organisation formed a topology-preserving map. The maps were used to create two-dimensional projections of higher dimensional data, in which the topology of the input space was preserved.

The resurgence of ANNs can be partly attributed to the discoveries of Hopfield who moved neural networks into the modern era of research (Freeman and Skapura 1992). Hopfield (1982) and Hopfield (1984) presented two models of associative memory, discrete Hopfield memory and continuous Hopfield memory using binary and continuous neurons, respectively. Both models featured recurrent networks that allowed the stable states of the network to be found under a process of parallel relaxation using an energy function minimisation rule. These stable states were related to the memory stored by the network. Another method used the Boltzmann distribution to find energy minima; the network was known as the Boltzmann machine and was a binary device. Ackley et al. (1985) described a learning rule for the Boltzmann machine.

The popular error BP learning algorithm was independently discovered by a number of researchers. First by Werbos (1974) and then almost simultaneously by Parker (1985), Le Cun (1986) and Rumelhart et al. (1986). Rumelhart et al. (1986) describes the generalised delta rule for learning in networks with hidden layers. This was a major development and allowed networks to increase in complexity beyond the perceptron with the addition of hidden layers.

LVQ networks were developed by Kohonen (1988) and are closely related to SOM. The networks attempt to define clear class borders for efficient classification.

RBF neural networks were developed by Poggio and Girosi (1989; 1990) from the mathematical framework of regularisation theory, which seeks to find a smooth approximating function.

Since these early articles there have been many other important developments in the field that it is perhaps impossible to choose which is still ground breaking research. There are now entire journals devoted to neural networks. Many articles published after the above papers

have modified and optimised the basic ideas contained in them. Much of the research in neural networks is now being conducted by computer scientists and has hence moved away from attempting to understand neurobiology and cognition towards refining their use as a computational tool.

The application of ANNs can be found in a diverse range of fields. Their ability to learn from examples is a major advantage for problems that have no clearly defined set of rules. An exponential increase in computer power has also affected the use of neural networks, allowing more parameters and large networks to be used. The first real-world applications of neural networks were most probably for image and speech recognition; Fukushima et al. (1983) used a neural network for recognising hand written numerals. Lippman (1989) and Trentin and Gori (2003) both investigated the use of neural networks for speech recognition. Neural networks can be used for financial analysis. Kim and Lee (2004) described using an incorporated genetic algorithm and neural network approach to predict the stock market. Jasic and Wood (2004) tested the predictive abilities of neural networks to forecast changes of major stock markets for profitability. ANNs can be applied to seemingly chaotic large scale dynamic systems. Maqsood et al. (2004) investigated the applicability of using an ensemble of neural networks to predict the weather. In the field of aerospace, Suresh et al. (2003) used a recurrent neural network to predict the lift coefficient of rotor blades at high angles of attack to investigate the dynamic stall effect. Reddy and Ganguli (2003) applied RBF neural networks to detect structural damage in a helicopter rotor blade. Breke et al. (1993) found optimal designs for aircraft wings under different dynamic and static constraints using neural networks trained with optimised design data. Civil engineering applications of neural networks include the forecasting of floods (Thirumalaiah and Deo 1998) and the flow for hydropower plants (Coulibaly et al. 2000). Basheer (2000) used neural networks to model the stress-strain behaviour of soils. Alsugair and Al-Qudrah (1998) developed neural network to assist inspectors assessing pavement maintenance. Lee and Lee (2004) classified crack types in digital pavement images with the use of several neural networks.

2.1.2. Applications in structural engineering

The first reported application of ANNs in the structural engineering field was made by Adeli and Yeh (1989) in which a perceptron network was used to design steel beams. Since then their use has grown extensively into a range of structural applications including analysis, design, optimisation, control, damage prediction and SHM. An comprehensive review of the

use of neural networks in civil engineering from 1989-2000 can be found in Adeli (2001). Although focusing mainly on the structural engineering and construction management fields the review also includes uses in environmental, water resources, traffic and geotechnical engineering.

Neural networks have been used in structural design for optimisation problems and computerisation of the design process. Unlike other artificial intelligence systems, neural networks learn from examples and design rules therefore do not need to be explicitly stated. Mukherjee and Deshpande (1995a) used a BP neural network to predict the optimal design of an RC beam, useful in the initial design phase. They stated that Rule-Based Expert Systems (RBES) lack learning and generalisation abilities and therefore their use was limited in design, unlike neural networks that can store expert knowledge and are able to generalise. Mukherjee and Deshpande (1995b) described using a combination of RBES and ANNs as a method of computerising the design process. The authors saw such a combination is suited to the design process that requires not only calculations but also engineering judgement, intuition, experience and creative abilities. The approach required the ANN to do the preliminary and detailed design, while the RBES did further processing and calculations. Adeli and Park (1995b) introduced a neural dynamics model for the optimal design of structures by using a stability function and introducing constraints on the solution. Adeli and Park (1995c) applied the optimisation method presented in Adeli and Park (1995b) to a minimum weight design of a space truss under various loads and constraints. Although a majority of research has used BP neural networks there are alternatives. Adeli and Park (1995a) outlined the applicability of CP neural networks in structural design whilst stating that BP algorithm was not suited for large networks due to its slow learning rate.

Mukherjee et al. (1996) predicted the buckling load of slender columns using a BP neural network trained from experimental data. Chuang et al. (1998) predicted the capacity of slender pin-ended reinforced concrete columns using neural networks trained from experimental data, an application useful in checking designs. Biedermann (1997) showed that neural networks could represent heuristic design knowledge and used a BP neural network to group structural members in 2D steel frames into design groups as an example. An insight into some of the issues with using neural networks was also given: the effects of network topology, data representation, distribution of training data and overtraining on the predictive ability of the network were discussed. Jenkins (2002) described a neural network based

iterative method for structural reanalysis using a plane truss and space truss as examples. The network was capable of dealing with a number of design changes including material, cross-section and load changes.

Structural control often involves complex nonlinear relationships and hence is suited to the application of neural networks. Bani-Hani and Ghaboussi (1998) investigated using a BP ANN to replace conventional control system theory in the control of an active tendon system. The authors concluded that neural networks were effective nonlinear controllers. Madan (2006) used BP ANNs for active control of a 8-DOF model with either an active mass damper or two active braces under earthquake excitation. Bani-Hani (2007) applied neural networks to control wind-induced vibration in an analytical model of a 72-storey building proposed for construction. The ANN was used to control an active tuned mass damper.

Other applications of ANNs directly related to the research presented in this report have been included in the relevant sections.

2.2. Structural Health Monitoring

This review of SHM literature has an emphasis placed on methods applied to civil infrastructure, although methods applied to aerospace, composite and non-destructive materials testing fields that are of particular interest have also been included. SHM methods can be broadly divided into several paradigms: vibration methods, visual inspection, and localised tests (Aktan et al. 2002). Vibration based methods appear to be the most promising and widely researched. Such methods rely on changes in the physical properties of structure i.e. stiffness, boundary conditions, damping and mass caused by damage to be represented in the dynamic response of the structure. Typically, vibration data, e.g. strains, displacements, velocities and accelerations, are transformed using various mathematical techniques in order to find a damage sensitive feature. Many methods employ a second mathematical tool, usually a statistical technique, to interpret changes in the value of this feature, providing damage classification into states or giving a quantitative measure of damage.

A comprehensive review of vibration based methods prior to 1996 can be found in Doebling et al. (1996). A companion study by Sohn et al. (2003) reviewed literature between 1996 and 2001. The review presented in this report has been divided into frequency domain methods, time domain methods, studies on the effects of environmental and operating conditions on

SHM and methods specifically using time series analysis techniques. The division of SHM methods into the frequency and time domain was broad and in some cases a degree of ambiguity may exist.

2.2.1. Frequency domain

Wu et al. (1992) used Fourier spectra of acceleration time histories and a BP ANN to detect damage in a simple linear 3-storey building model. The Fourier spectra were used as the ANN input while the output was the damage at each storey.

Worden (1997) introduced the concept of novelty detection, a unsupervised damage detection methodology. Transmissibility data obtained in the undamaged condition was used as input into an Auto-Associative ANN. When damage was present the ANN was no longer able to reproduce the input at the output layer and this error indicated the presence of damage.

Doyle and Fernando (1998) classified acoustic frequency data obtained from a composite panel using BP ANNs and LVQ into four different states. Performance of both network types was found to be similar.

Zang and Imregun (2001) used Frequency Response Functions (FRFs) reduced by Principle Component Analysis (PCA) as inputs into a BP ANN. Without the reduction using PCA the use of ANNs would be impractical, as ANN training would require a large computational effort. The authors applied the method to detecting damage in a railway wheel.

Sohn and Law (2001) investigated the use of Ritz vectors in detecting damage in a bridge structure. The Ritz vectors were extracted from a flexibility matrix constructed from vibration data. The authors showed that by selecting appropriate load patterns, Ritz vectors were more sensitive to damage than modal vectors.

Demetriou and Hou (2003) investigated two damage detection methods, a wavelet based and RBF based approach, applied to a 1 and 3-DOF mechanical system. Damage was simulated by a broken spring, causing an abrupt loss in stiffness. Although both approaches could detect a change in the system caused by the loss in stiffness, only the RBF approach could estimate the stiffness.

Ni et al. (2006) investigated seismic damage detection in a 1:20 scale model of a 38-storey building. The building was subjected to several levels of excitation by a shake table. These different levels of excitation corresponded to the damaged states of the building. FRFs were obtained from ambient vibration tests when the building was in its healthy and damaged states and subsequently compressed by PCA. The authors stated that this served two purposes, dimensionality reduction and noise elimination. The principal components were used as ANN inputs and prediction error was used as a damage sensitive feature. Identification of damage location was achieved by using multiple ANNs, each monitoring several stories.

Hou et al. (2006) proposed a wavelet-based method for detecting damage in structures subjected to seismic excitation. Wavelet analysis and the Hilbert Transform were used to identify modal parameters online from vibration response data. Changes in mode shapes were used to detect damage.

2.2.2. Time domain

Non-parametric system identification methods using ANNs have been widely researched for SHM. Nakamura et al. (1998) studied a 23m high 7-storey steel frame damaged in the 1995 Japanese Kobe earthquake. Although the building was not instrumented during the earthquake, the authors carried out ambient vibration measurements of the building in its damaged and repaired states. The repairs were considered to return the structure to its original undamaged or healthy state. An ANN was used to relate the interstorey displacement to the interstorey restoring force when the building was undamaged. Prediction errors between the ANN and measurements were used to identify damage in the structure. Due to its non-parametric nature the method was not capable of determining which particular member(s) were damaged and the extent of this damage.

Wu et al. (2002) extended the study of Nakamura et al. (1998) and described a decentralised parametric identification method using ANNs for the detection of damage in MDOF structures. The MDOF structure was broken into substructures with a smaller number of DOF. An ANN was trained for each substructure to predict the restoring force given the interstorey displacement and velocity as inputs. The prediction error between the ANN and measurements was used as an input into a second ANN. This network quantified damage as a change in stiffness.

Huang et al. (2003) used an ANN to identify changes in the buildings dynamic response and corresponding structural damage. The authors trained the network to predict undamaged response of the structure in minor earthquakes, in which the structure behaved linearly. Errors between the prediction of the network and the measured response occurred when the structure behaved non-linearly.

A recent study by Jiang and Adeli (2007) used a fuzzy Wavelet Neural Network (WNN) to detect damage in a 38-storey concrete model caused by seismic excitation. The structure was divided into smaller substructures and the responses of each substructure and ground excitation were formulated as a state-space model. The state vector was used as input into the WNN, while the output was the estimated response for each substructure. Prediction error was used as a feature to estimate damage from a pseudospectrum.

Godin et al. (2004) investigated the use of a supervised and two unsupervised pattern recognition techniques, k -Nearest Neighbour, k -means and SOM, for classification of different damage mechanisms in composites. The authors used several properties of acoustic emission signal as damage sensitive features. Results showed similar performance for k -Nearest Neighbour and SOM.

Alvandi and Cremona (2006) reviewed and evaluated the performance of several vibration based SHM methods. These were the mode shape curvature, changes in flexibility matrix, flexibility curvature and strain energy methods. The authors concluded that the strain energy method was the most efficient technique reviewed.

Methods have also been developed that use attractor analysis and chaotic excitation to detect damage. Nichols et al. (2003a) developed an attractor based nonlinear damage detection method. By exciting the structure, in this case a beam, with a tuned chaotic signal, the structural response was ensured to be low dimensional. Low dimensionality is very important condition for attractor methods. Attractors were obtained from the undamaged structure and used to predict the response of the structure. Prediction error indicated the presence of damage.

Nichols et al. (2003b) investigated two different measures of the attractor dimension as a damage feature: the Takens estimator and correlation dimension. A hypothesis test on the means of these features was used to determine if the response was from a damaged or undamaged structure. Experiments on a beam excited by a chaotic signal showed that the Takens estimator was a better choice than the correlation dimension.

Casciati and Casciati (2006) investigated the use of Lyapounov exponents and the Lyapounov dimension for damage detection in nonlinear systems. Changes in the values of these parameters were used to detect damage in an arch structure. Damage localisation was achieved by analysing subsets of the data.

Nichols et al. (2006a) investigated the concepts of information theory for nonlinear damage detection. Two information metrics were chosen: time-delayed mutual information and time-delayed transfer entropy. When the structure, a 5-DOF model, changed from linear to nonlinear behaviour, analysis of the two metrics allowed the degree of nonlinearity to be assessed.

Nichols et al. (2006b) developed a method for detecting non-stationarities in time series data using multivariate Recurrence Quantification Analysis (RQA). The approach was applied to a FEM model of a plate with damage introduced by a cut. Extracted RQA features were found in some cases to be more sensitive to damage than natural frequencies.

Instead of employing various mathematical transformations of vibration data to obtain damage sensitive features, some studies have attempted to track structural properties e.g. stiffness, damping and hysteretic properties, online. Smyth et al. (1999) used an adaptive least-squares approach to identify the hysteretic properties in MDOF models.

Cooper and Worden (2000) developed an online method for tracking stiffness, damping and mass of a structure based on the dynamic equation of motion. The forgetting factor and adaptive forgetting factor recursive algorithms were investigated. The authors used the proposed method to track the parameters of various SDOF models.

Chase et al. (2005) used an adaptive recursive least-squares filter to identify changes in stiffness in 4-DOF and 12-DOF analytical models of the ASCE Phase II Experimental SHM Benchmark Structure (ASCE Structural Health Monitoring Committee).

Yang et al. (2006) used an extended Kalman filter with adaptive tracking to identify stiffness, damping and hysteretic parameters in linear 1-DOF and nonlinear 2-DOF structures. Yang and Lin (2005) proposed a recursive least-squares adaptive tracking technique for time varying systems. The authors stated that this adaptive method was suited to systems in which the parameters changed abruptly. The approach was applied to identifying and tracking stiffness and damping in several linear and nonlinear structures. Yang et al. (2007) extended the previous work to include the case of unknown input. Stiffness, damping and hysteretic properties were identified in a 12-DOF analytical model of the ASCE Phase II Experimental SHM Benchmark Structure and a nonlinear hysteretic 2-DOF structure.

Sohn et al. (2007) applied the concept of time reversal using Lamb waves for damage detection in thin composite plates. Time reversibility is based on linear reciprocity of elastic waves. Damage causes defects in the medium and these nonlinearities violate time reversibility. A damage feature quantifying the degree of reversibility violation was used to detect the presence of damage with the use of extreme value statistics.

2.2.3. Environment effects and operating conditions

One of the most problematic aspects of implementing a SHM system in practice is the effect of changing environmental and operating conditions. Very few researchers have successfully incorporated or researched the consequences of these effects on their proposed SHM methodology. Such an investigation requires long term monitoring of a built structure and hence lies outside the scope of this research. The results of this study should be viewed with this in mind.

Environment effects, e.g. temperature, wind, humidity, etc., cause differences in the response of a structure. Sohn et al. (1999) showed that the changes in modal parameters caused by environmental factors can in fact be larger than the changes caused by noticeable damage. The authors presented an adaptive filter approach in which damage was detected from changes in natural frequencies. The proposed approach incorporated the changes in natural frequencies due to changes in temperature. Peeters et al. (2001) used an Autoregressive

eXogeneous input (ARX) model to relate natural frequencies to temperature. Prediction error was used to detect damage.

Farrar et al. (2000) measured the change in the natural frequencies of the Alamosa Canyon Bridge over a 24-hour time period. The changes were assumed to be mainly due to changes in the temperature. A 5% variation in the first mode frequency was observed over this time period. Operating conditions can also affect the response of a structure. In the same study, tests were conducted to evaluate the changes in natural frequencies caused by vehicle weight. Four cars were parked on the bridge, adding a total of 99kN to the concrete span weight of 703kN. The results were not in agreement with theory and the authors concluded that some complex interaction with the car suspension must have occurred.

Kullaa (2002; 2004) used factor analysis to detect damage in structures such as a wooden truss and a crane under different temperature and moistures, and loads and configurations, respectively. Factor analysis is a statistical technique used to attribute variability amongst a large set of observed random variables to a smaller set of unobserved variables. The author later considered using the missing data concept to achieve separation of feature changes resulting from damage from those caused by the operating environment (Kullaa 2005).

Sohn (2007) reviewed studies on the effects of changing environmental and operating conditions on real structures. Data normalisation techniques were presented as an effective approach for removing the contributions of environmental and operating variations.

2.2.4. Time series methods

The use of time series analysis techniques for SHM has so far received limited attention from researchers. However, these methods are promising and may overcome some of the difficulties faced by other SHM methods, e.g. sensitivity to environmental effects and operating conditions. Time series techniques, originally developed for analysing long sequences of regularly sampled data are inherently suited to SHM. Also, time series methods are immediately applicable to the online detection of damage using recursive identification techniques.

In a pioneering study by Sohn et al. (2000), Autoregressive (AR) models were used to fit the dynamic response of a concrete bridge pier. By performing statistical control chart analysis

on the AR coefficients the authors were able to distinguish the responses from damaged and undamaged systems. However, no attempt was made to locate and quantify damage. In a later study by Sohn et al. (2001) a similar methodology was applied to detecting damage in a fast surface-effect naval patrol boat.

Several studies have used the residual error of time series models as damage sensitive features. Sohn and Farrar (2001) used a AR-ARX modelling approach to detect damage in an 8-DOF system. By modelling a sampled signal with an AR model and comparing the coefficients to a reference database of known AR coefficients, the authors were able to reproduce the sample signal using closest AR coefficients in the database. A ratio of the standard deviations of the residual error obtained from using ARX model for the sample and reference signal was used as a damage sensitive feature.

Fugate et al. (2001) used AR models to fit the response of a concrete bridge pier, same structure as in Sohn et al. (2000). Residual errors between the response predicted by the AR model and the actual measured response were used as damage sensitive feature. Control chart analysis was used to monitor changes in the mean and variance of the feature and indicate the presence of damage.

Manson et al. (2001) created two-dimensional projections of AR coefficient data using PCA and Sammon mapping. AR coefficients were obtained from fitting acoustic emission signal time series of a box girder. The authors attempted to identify clusters in the projections corresponding to different emissions.

Moyo and Brownjohn (2002) studied continuous strain data from a monitoring system installed on the Singapore-Malaysia Second Link Bridge. They applied intervention analysis to assess the effects of sudden strain changes on the subsequent strain state in the bridge.

Omenzetter et al. (2004) studied multi-channel strain data collected from the Singapore-Malaysia Second Link Bridge. The authors developed a method of detecting and locating abrupt events in the strain data. A vector ARMA model was used to fit wavelet coefficients from earlier wavelet analysis of the strain time histories. Multivariate outlier analysis was used to identify and locate the events causing the abrupt changes in the strain data.

In a subsequent study, Omenzetter and Brownjohn (2006) investigated the use of a vector seasonal Autoregressive Integrated Moving Average (ARIMA) model for the same data. The seasonal time series model was used to account for periodic strain variation caused by the temperature cycle. The coefficients of the ARIMA model were identified online using the extended Kalman filter. Changes in these coefficients were used to detect sudden events experienced by the structure.

Nair et al. (2006) used an Autoregressive Moving Average (ARMA) time series to model the vibration signal from the ASCE Phase II Experimental SHM Benchmark Structure (ASCE Structural Health Monitoring Committee). The authors defined a damage sensitive feature used to discriminate between the damaged and undamaged states of the structure based on the first three AR coefficients. It was found that statistical mean of this feature was different for damaged and undamaged structures. A *t*-test was used to determine if the structure was damaged. Location of damage was achieved by introducing another set of features found to increase from a baseline value when damage had occurred near to the sensor location.

Nair and Kiremidjian (2007) investigated Gaussian Mixture Modelling, an unsupervised pattern recognition technique to model the feature vector. ARMA models were fitted to acceleration data obtained from a 12-DOF analytical model of the ASCE Phase II Experimental SHM Benchmark Structure. Selecting the first three AR coefficients for analysis only, damage detection was achieved by determining the number of patterns in the data set using a statistical measure. The extent of damage was shown to correlate well with the Mahalanobis distance between undamaged and damaged patterns.

Gul et al. (2007) used AR coefficients obtained from a laboratory steel beam to classify varying support conditions using a clustering algorithm or a multivariate statistical technique.

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CHAPTER 3

EXPERIMENTAL STRUCTURES, THEIR TESTING AND IDENTIFICATION OF MODAL PROPERTIES

The damage detection methods investigated in this research project were applied to, and verified on, experimental data collected from three laboratory structures: a 3-storey bookshelf structure, a RC column, and the ASCE Phase II Experimental SHM Benchmark Structure. The first two structures were specially built and tested for this project, while data from the third one is freely available to researchers from the ASCE SHM Committee via Internet (ASCE Structural Health Monitoring Committee). In this chapter, a description of the three experimental structures is given. A full modal testing programme determining natural frequencies, damping ratios and mode shapes in the healthy or undamaged state was conducted on the 3-storey bookshelf structure and RC column as part of the project in order to understand the dynamics of these systems and the results are presented herein. Experimental modal analysis techniques underpin system identification and a description of the methods used is also given. Two good reference texts on modal testing for the interested reader are Heylen et al. (1997) and Ewins (2000).

3.1. 3-storey bookshelf structure

The 3-storey bookshelf structure was approximately 2.1m high and constructed from aluminium angles and stainless steel floor plates joined together with bolted aluminium brackets as shown in Figure 3.1a,b. Design of the structure followed guidelines in the New Zealand Aluminium Structures Code (Standards New Zealand 1997) and design loads were determined from computer simulations. A number of factors were considered in the design of the structure. One critical factor was the ability to change the lateral stiffness of each storey

independently without permanently damaging the structure. This was achieved by allowing the columns to be easily replaceable using brackets. The limitations of the performance of the shake table were also considered as an important factor. Consequently, the natural frequencies of the structure were kept low to ensure that all modes could be excited.

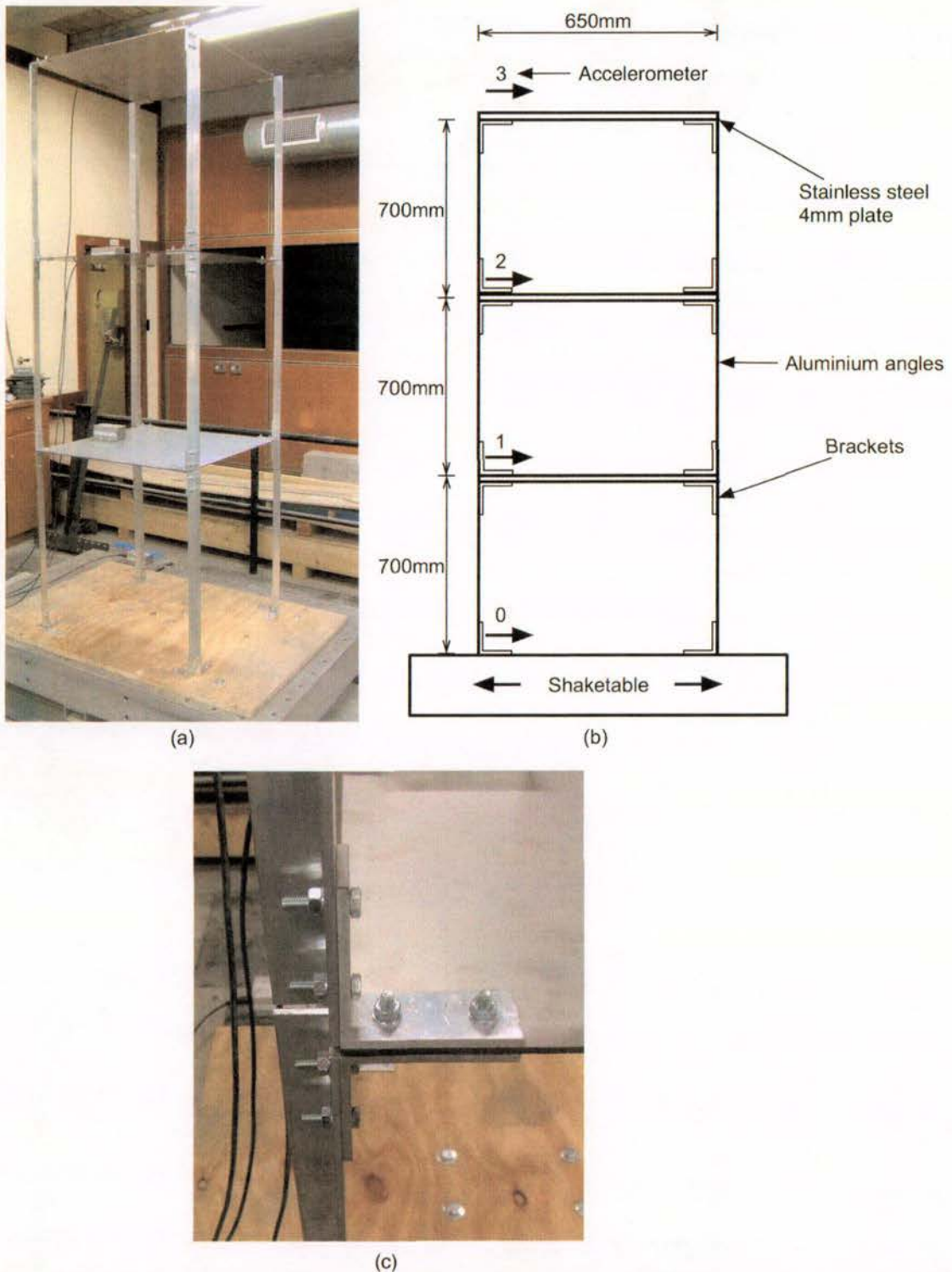


Figure 3.1. 3-storey bookshelf structure mounted on shake table: (a) general view, (b) diagram of accelerometer locations and external dimensions, and (c) detail of column-plate joint.

The stainless steel plates were 4.0mm thick and 650mm × 650mm square. Two equal angles sizes were used as columns, 30mm × 30mm with a thickness of either 3.0mm or 4.5mm for the damaged and undamaged columns, respectively. The columns were 0.7m long and fastened with two bolts at each end to aluminium brackets. These brackets were attached with two 6.0mm bolts to the floor plates, see Figure 3.1c. The brackets were 30mm wide and 4.5mm thick. At the base, additional brackets in the direction orthogonal to motion provided extra restraint. The whole structure was mounted on a 20mm plywood sheet bolted with 10mm bolts to the shake table.

The shake table used in the experiments is located in the Test Hall of the Department of Civil and Environmental Engineering, the University of Auckland. The shake table is capable of producing motion in one direction only. An actuator and servo valves use hydraulic pressure provided by a pump (Figure 3.2a) to move the table. A Proportional Integral Derivative (PID) controller (Figure 3.2b) is used to control the motion of the table and receives feedback from the Direct Current Displacement Transducer (DCDT) on the displacement of the table. An acquisition box (Figure 3.2b) provides the physical interface between the sensors and the data-logging card in the computer. A computer program converts acceleration records to displacement commands and simultaneously issues and records measurements from the table.

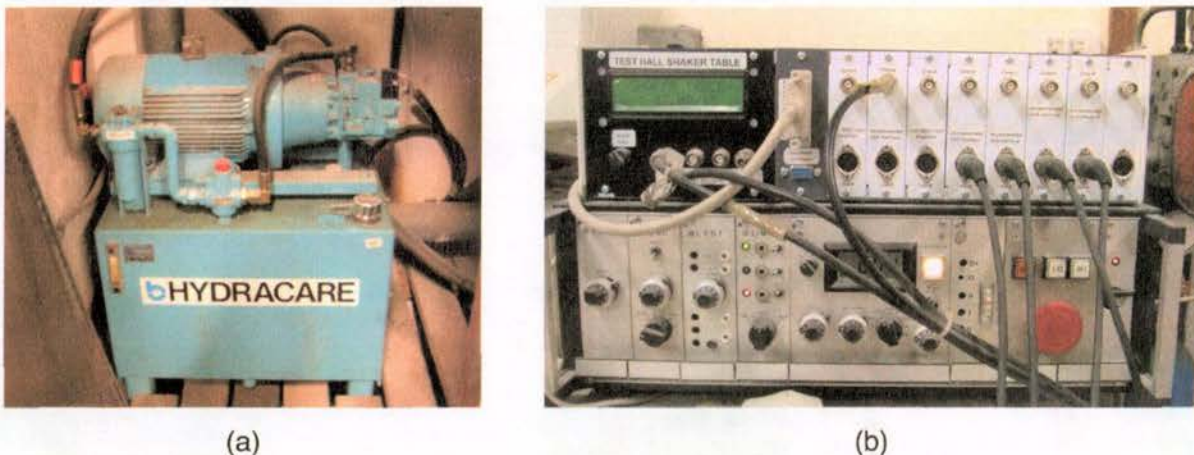


Figure 3.2. Shake table equipment: (a) pump, and (b) PID controller (below) and acquisition box (above).

The structure was instrumented with four 2.5Vg^{-1} uniaxial accelerometers, one for measuring the table acceleration and one for each storey as shown in Figure 3.1b. Accelerations were measured in the direction of ground motion at a sampling rate of 400Hz using a computer

fitted with a data-logging card. MATLAB was used to both write and filter the data. All data was filtered with a zero phase shift 50Hz low pass filter. Afterwards the data was decimated by a factor of four. This reduced the original 400Hz signal down to 100Hz. The decimate procedure implemented in MATLAB uses an eight order Chebyshev Type I low pass filter with cut-off frequency $(0.8/R) \times (F_s/2)$, where F_s is the initial sampling frequency and R is the decimate factor, before resampling the data. The decimate process improved the quality of the data by further removing high frequency noise.

3.1.1. Modal analysis

The primary purpose of the test program was to find the natural frequencies, modal damping ratios and mode shapes of the structure in an undamaged state from forced vibration tests. These tests were conducted over approximately a 2-week period in July 2006. The forced vibration tests consisted of frequency sweep tests, free vibration decay tests and Gaussian white noise ground excitation.

Frequency sweep tests using a sine forcing function at different forcing frequencies allowed acceleration response curves to be plotted, natural frequencies estimated and the modal damping ratios estimated using the half-power bandwidth method. Mode shapes were also extracted from the acceleration data. Free vibration decay tests were conducted to estimate the modal damping ratios using the popular logarithmic decrement method. Gaussian white noise excitation was used to construct Frequency Response Functions (FRF) and a state-space model of the structure. From the state-space model, natural frequencies, modal damping ratios and mode shapes were estimated. A description of these methods and a discussion of the results are given below.

3.1.1.1. Frequency Response Functions

To get an initial estimate of the natural frequencies, the FRF for the 3rd storey of the structure was calculated using Welch's method with a Hanning window and no overlapping. Input into the shake-table was 20s of white noise. Following the development in (Ewins 2000), for a random discrete time signal the FRF can be computed using

$$H(\omega) = \frac{X(\omega)}{F(\omega)} \quad (3.1)$$

where $X(\omega)$ and $F(\omega)$ are the Discrete Fourier Transforms (DFT) of the response and input excitation, respectively. When both the response and input excitation are random processes, an alternative form must be used. Considering the input excitation $f(t)$, the autocorrelation function R_{ff} at time τ can be calculated using

$$R_{ff}(\tau) = E[f(t)f(t+\tau)] \quad (3.2)$$

where E is the expectation operator. The Power Spectral Density (PSD) is the Fourier transform of the autocorrelation function and is defined as

$$S_{ff}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ff}(\tau) e^{-i\omega\tau} d\tau \quad (3.3)$$

Given the structural response $x(t)$, the cross-correlation function can be calculated using

$$R_{xf}(\tau) = E[x(t)f(t+\tau)] \quad (3.4)$$

The Cross-Spectral Density (CSD) is the Fourier transform of the cross-correlation function and can be calculated using

$$S_{xf}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xf}(\tau) e^{-i\omega\tau} d\tau \quad (3.5)$$

The CSD is generally a complex function and has the following conjugate property:

$$S_{xf}(\omega) = S_{fx}^*(\omega) \quad (3.6)$$

Two alternative formulations of FRF be can proposed:

$$H_1(\omega) = \frac{S_{fx}(\omega)}{S_{ff}(\omega)} \quad (3.7)$$

$$H_2(\omega) = \frac{S_{xx}(\omega)}{S_{xf}(\omega)} \quad (3.8)$$

H_1 is most commonly used. The DFT assumes that the signal is periodic or complete over the finite length of the observation time. If this assumption is false, which is often the case for

random signals, spectral leakage occurs and energy at frequencies actually contained in the signal will 'leak' into the spectral lines surrounding these frequencies. This problem can be mitigated by the application of windowing functions. Windowing involves imposing a profile, usually a time domain function $w(t)$, on the signal prior to the DFT:

$$x'(t) = x(t)w(t) \quad (3.9)$$

A commonly used windowing function is the Hanning window:

$$w(t) = 0.5 \left(1 + \cos \left(\frac{2\pi t}{T} \right) \right) \quad (3.10)$$

where t is the time step and T is the length of the record. In some cases rescaling is required to compensate for changes caused by the application of the windowing function. However, if both the input and response are subjected to the same window, calculation of the FRF curve does not require rescaling.

The FRF obtained from the 3rd storey of the 3-storey bookshelf structure shows three definite peaks at 1.95Hz, 5.46Hz and 8.59Hz corresponding to the three modes of the structure, see Figure 3.3.

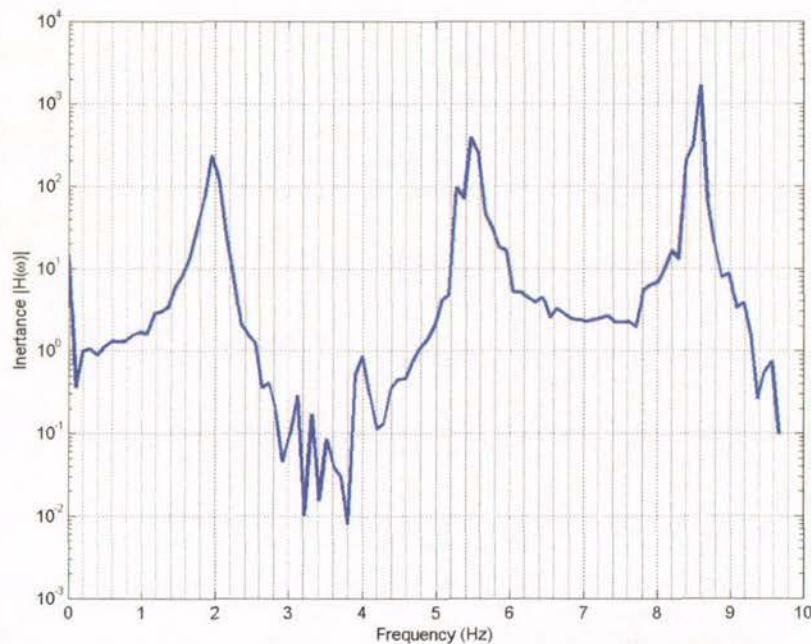


Figure 3.3. Acceleration FRF for the 3rd storey of 3-storey bookshelf structure.

3.1.1.2. Frequency sweep tests

Another method for identifying natural frequencies is to record the maximum steady state acceleration when the structure is excited by a periodic forcing function with a known frequency and amplitude. From these results an acceleration frequency response curve can be plotted. The peak responses in the curve are the natural frequencies of the structure. Frequency sweep tests from 1.5-2.5Hz, 5-6Hz and 8-9Hz with amplitudes of ground motion of 0.01g, 0.05g and 0.25g, respectively, were conducted using a sine forcing function over 20s intervals. The maximum acceleration was measured at the 3rd storey and taken as an average of 10 maximum peaks once the transient response had decayed. Figure 3.4 shows the acceleration response curve for the frequencies between 1.5 and 2.5Hz; at 1.87Hz there is a definite peak that corresponds to the 1st natural frequency. Similar graphs were obtained for the 2nd and 3rd natural frequencies and are shown in Figures 3.5 and 3.6. The graphs show that the 2nd and 3rd frequencies are 5.42Hz and 8.43Hz, respectively. These results are similar to the values obtained previously from the FRF determined via white noise excitation.

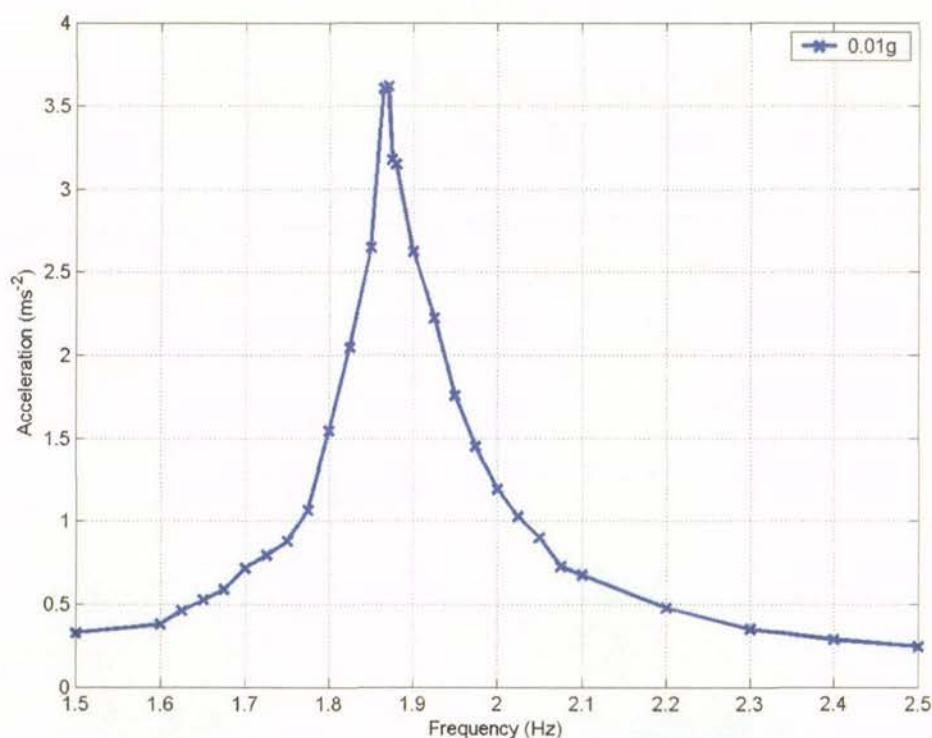


Figure 3.4. Acceleration response curve for 3-storey bookshelf structure at the 3rd storey showing the 1st natural frequency.

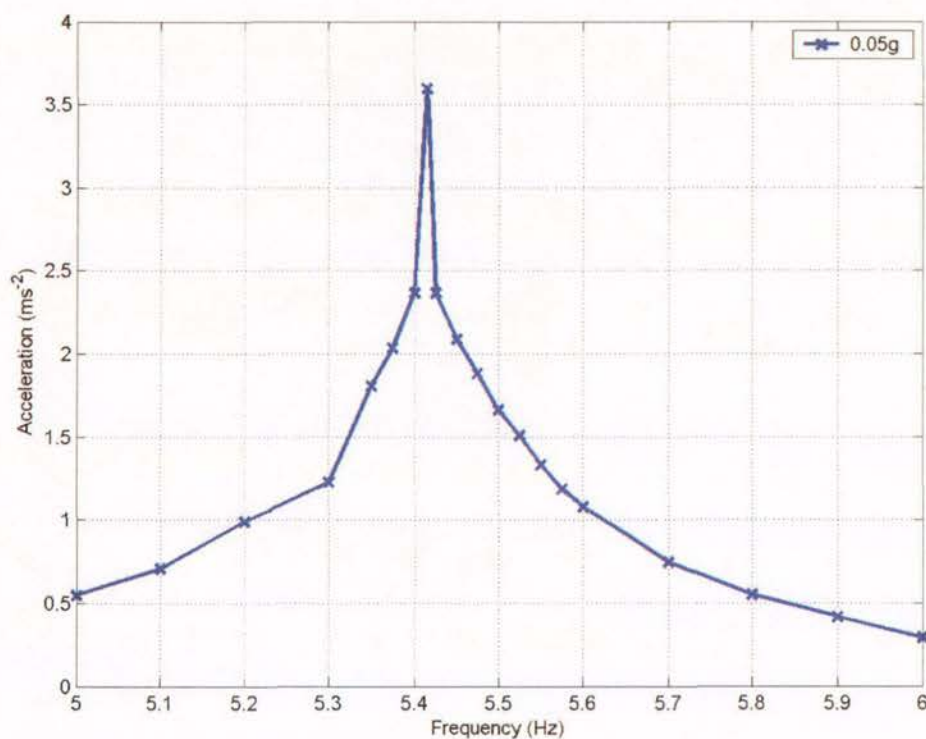


Figure 3.5. Acceleration response curve for 3-storey bookshelf structure at the 3rd storey showing the 2nd natural frequency.

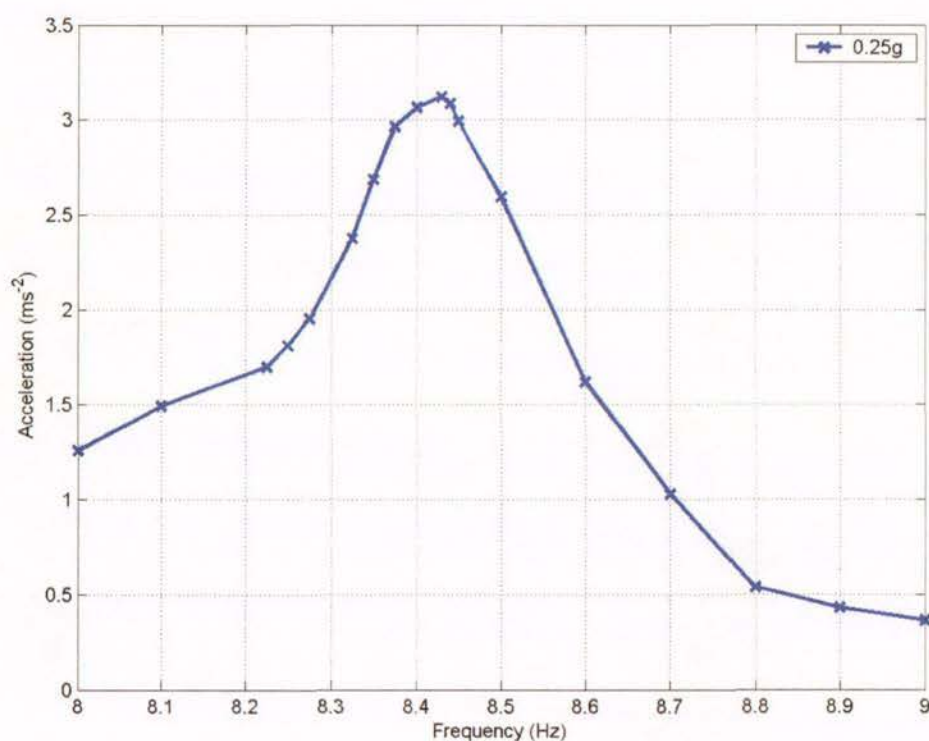


Figure 3.6. Acceleration response curve for 3-storey bookshelf structure at the 3rd storey showing the 3rd natural frequency.

3.1.1.3. State-space system identification

The natural frequencies, damping ratios and mode shapes can also be identified from identification of a discrete-time state-space model. A discrete state-space model at an arbitrary time step k can be written as

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \quad (3.11)$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k \quad (3.12)$$

where $\mathbf{x}_k$, $\mathbf{u}_k$ and $\mathbf{y}_k$ are respectively the state, input and output vectors at time k . The matrices in Equations (3.11) and (3.12) are referred to as follows: $\mathbf{A}$ is the state matrix, $\mathbf{B}$ is the input matrix, $\mathbf{C}$ is the output matrix, and $\mathbf{D}$ is the feedthrough matrix. These system matrices need to be determined in the identification process. In this case, the output vector was the three floor accelerations and the input vector was the table acceleration. The Prediction Error Method (PEM) identification algorithm implemented in the system identification toolbox in MATLAB (Ljung 2006) was used to estimate the system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$, where $\hat{\cdot}$ denotes that the matrices are estimated quantities. This method attempts find the state-space matrices such that the responses predicted via Equations (3.11) and (3.12) have the smallest overall error when compared to actual, measured responses. In Figure 3.7, the identified structural response has been plotted with the actual response obtained from measured acceleration data for a 20s record. The responses are very similar for the 2nd and 3rd stories, while the 1st storey shows some discrepancy.

The natural frequencies and damping ratios can be extracted from the imaginary and real parts of the eigenvalues of $\hat{\mathbf{A}}_c$, the continuous time counterpart of matrix $\mathbf{A}$. Conversion to a continuous- time matrix can be achieved using

$$\hat{\mathbf{A}}_c = \frac{\ln(\hat{\mathbf{A}})}{\Delta t} \quad (3.13)$$

where $\ln$ is the natural logarithm and Δt is the sampling interval. The natural frequency ω_j and damping ratio ξ_j for the j^{th} mode can be extracted from eigenvalue decomposition:

$$\hat{\mathbf{A}}_c = \Psi \Lambda \Psi^{-1} \quad (3.14)$$

where the diagonal matrix Λ stores the eigenvalues or system poles λ_j related to natural frequencies and damping ratios as follows:

$$\lambda_j = -\xi_j \omega_j \pm i \omega_j \sqrt{1 - \xi_j^2} \quad (3.15)$$

where i is the imaginary unit. Mode shapes at the measurement locations can be calculated from

$$\Phi_c = C\Psi \quad (3.16)$$

The mode shapes Φ_c obtained in Equation (3.16) are in general complex and the following transformation (Friswell and Mottershead 1995) can be applied to convert them to real mode shapes Φ_r :

$$\Phi_r = \text{Re}(\Phi_c) + \text{Im}(\Phi_c) \text{Re}(\Phi_c)^+ \text{Im}(\Phi_c) \quad (3.17)$$

where Re and Im are the real and imaginary parts, respectively, and $^+$ denotes the pseudoinverse.

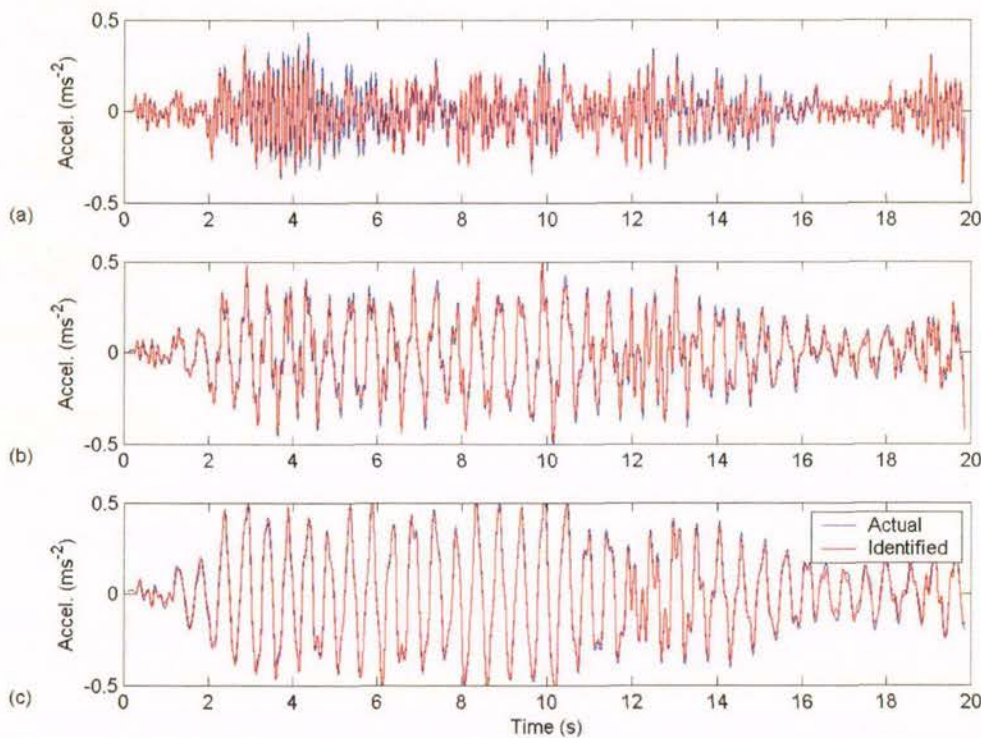


Figure 3.7. Actual and identified state-space response for 3-storey bookshelf structure: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

The modal parameters were estimated from five records, four 10s long and one 20s long. In all cases, the excitation or input was Gaussian white noise. Table 3.1 shows the estimated natural frequencies f and damping ratios ξ obtained from the five intervals. The lower and upper bounds correspond to two standard deviations or 95% confidence levels. The damping ratios were much harder to estimate and had greater uncertainty than the natural frequencies. The modes shapes were estimated from the 20s record only. Table 3.2 gives the normalised mode shapes for a maximum of 1.

3.1.1.4. Free vibration decay method

To estimate the damping ratio at each natural frequency, the logarithmic decrement method (Ewins 2000) was applied to free vibration decay tests when the structure was initially excited by a sine forcing function for 25s at one of the natural frequencies. The free vibration decay was recorded over a 25s interval after the initial excitation had ceased. Figure 3.8 shows the free vibration decay of the 3rd storey when excited in the 1st mode.

The damping ratio ξ can be calculated from the ratio of two positive or negative peaks of the displacement time history u_t separated by n vibration cycles:

$$\xi = \left(\frac{1}{2n\pi} \right) \ln \left(\frac{u_t}{u_{t+nT}} \right) \quad (3.18)$$

where T is the period. In this case the displacement was taken at the 3rd storey, n was set at 20 and the average of the results for the positive and negative peaks was taken as ξ . Alternatively, an exponential curve of the form

$$y = a \exp(-bt) \quad (3.19)$$

where a is a constant and b is the damping ratio, can be fitted to the positive or negative peaks of the displacement record using the least squares method. As before, the displacement was taken at the 3rd storey, the exponential function was fitted over 20 cycles and an average of the exponential curve parameters for the positive and negative peaks was taken.

The two methods gave identical values of damping for the 1st and 2nd modes of 1.1% and 0.9%, respectively. There was a small difference for the 3rd mode with the damping ratio

identified as 1.3% from the amplitudes of response peaks and 1.5% from the exponential curve fit.

Table 3.1. 3-storey bookshelf structure frequencies and damping ratios obtained from state-space model.

Mode	f (Hz)	ξ (%)
1 st	1.928±0.006	0.6±0.2
2 nd	5.52±0.02	0.6±0.2
3 rd	8.55±0.04	0.8±0.4

Table 3.2. 3-storey bookshelf structure normalised mode shapes obtained from state-space model.

Storey	Mode 1	Mode 2	Mode 3
1 st	0.18	0.64	1.00
2 nd	0.69	1.00	-0.48
3 rd	1.00	-0.88	0.16

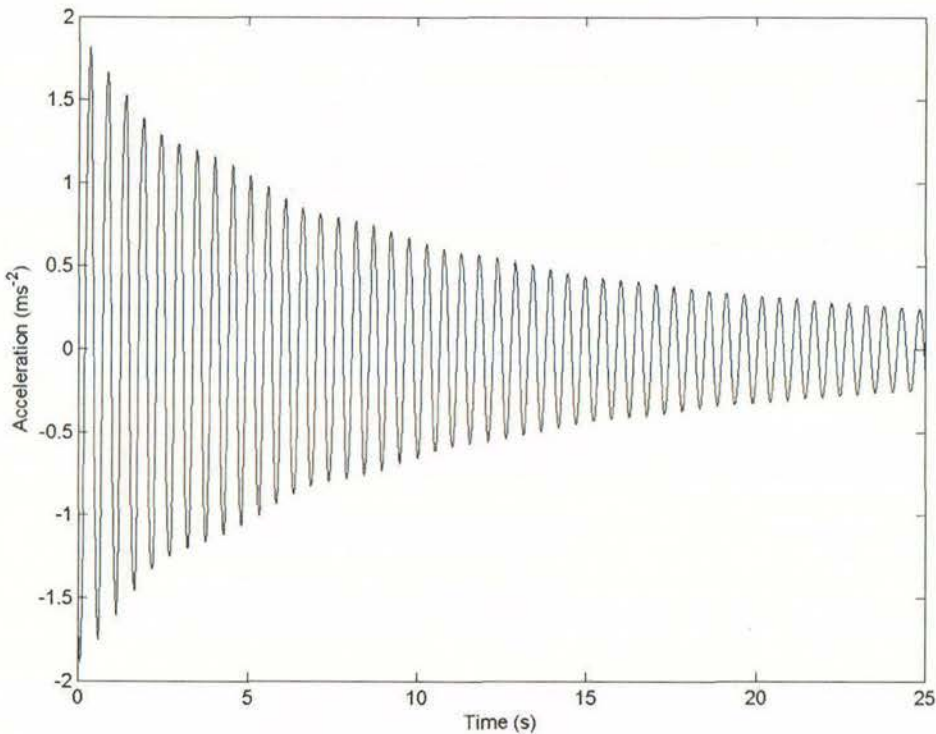


Figure 3.8. 3-storey bookshelf structure free vibration decay of the 3rd storey when excited in the 1st mode.

3.1.1.5. Half-power bandwidth method

The half-power bandwidth method (Heylen et al. 1997) is a popular and simple method for calculating damping ratios directly from the frequency response curve. An estimate of the damping ratio can be obtained from examination of the acceleration response curve as shown

in Figure 3.9. The frequencies f_1 and f_2 at which the frequency response is $1/\sqrt{2}$ of the peak value can be used to calculate the damping ratio using

$$\xi = \frac{f_2 - f_1}{f_2 + f_1} \quad (3.20)$$

At these frequencies the power is half the peak value hence the name of the method. A good estimate of the damping ratio requires good resolution of the response curve around the half-power frequencies. In this case, the resolution was sufficient to make a reasonable estimate, see Figures 3.4-6. The results were 1.6%, 0.3% and 1.4% for the 1st, 2nd and 3rd mode, respectively. The results were higher than the estimates from the free decay method with the exception of the 2nd mode. This lower than expected 2nd mode damping ratio is most likely due to the procedure used to construct the response curve and experimental error.

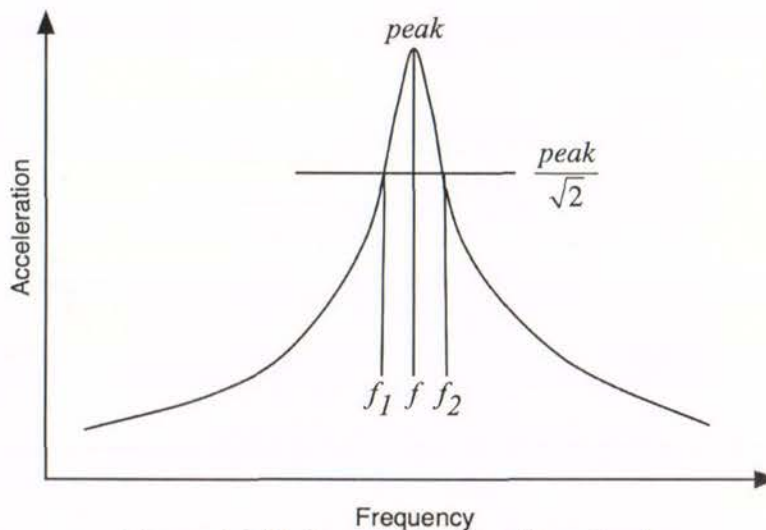


Figure 3.9. Half-power bandwidth method.

3.1.1.6. Mode shapes from acceleration data

By examining the acceleration data at the natural frequencies an estimate of the mode shapes could be obtained. An average of 10 consecutive positive peaks from the acceleration response, when the structure was excited with a sine forcing function at the natural frequency, was used to estimate the mode shape. The mode shapes were normalised for a maximum response of 1. The results are given in Table 3.3.

Table 3.3. 3-storey bookshelf structure mode shapes extracted from acceleration data.

Storey	Mode 1	Mode 2	Mode 3
1 st	0.24	-0.56	1.00
2 nd	0.74	-0.91	-0.54
3 rd	1.00	1.00	0.17

3.1.2. Discussion and summary

Natural frequencies for the 3-storey bookshelf structure have been accurately determined from various different methods and the results have been summarised in Table 3.4. A high level of agreement between the different methods used can be noticed, particularly for the FRF and state-space results. The state-space results are probably the most accurate because an average of several results was taken. Damping ratios were difficult to estimate due to the small amount of damping observed. The damping ratios obtained using different methods, listed in Table 3.5, appear to agree less between the different methods than the results achieved for the natural frequencies. Mode shapes obtained from the state-space model and the acceleration data were in good agreement. Figure 3.10 shows a graphical comparison between the results of the two methods.

Table 3.4. Summary of natural frequencies for 3-storey bookshelf structure obtained from different methods.

Mode	f (Hz)		
	FRF	Acceleration response curve	State-space
1 st	1.95	1.87	1.928
2 nd	5.46	5.42	5.517
3 rd	8.59	8.43	8.548

Table 3.5. Summary of damping ratios for 3-storey structure obtained from different methods.

Mode	ξ (%)			
	State-space	Peak to peak ratio	Exponential fit	Half-power
1 st	0.6	1.1	1.1	1.6
2 nd	0.6	0.9	0.9	0.3
3 rd	0.8	1.3	1.5	1.4

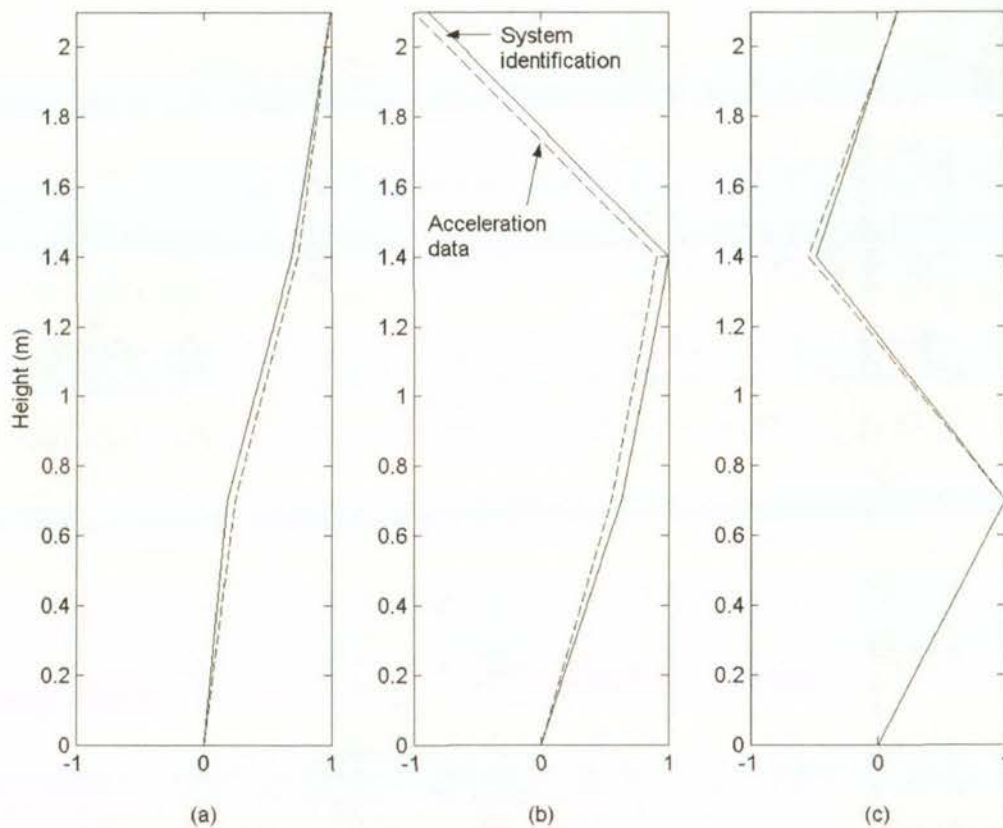


Figure 3.10. 3-storey bookshelf structure mode shapes obtained from the state-space model and acceleration data: (a) 1st mode, (b) 2nd mode, and (c) 3rd mode.

3.2. RC column

The cantilever RC column is shown in Figure 3.11. It was 1.1m high and had a 120mm $\times$ 120mm cross-section. The footing was 600mm $\times$ 600mm with a depth of 150mm. Principal reinforcement in the column was provided using 4-D10 bars. Wood planks were placed across the footing and bolted to holes in the concrete floor to fix the footing. An APS Dynamics ELECTRO-SEIS 400 Linear Mass Shaker was bolted to a plywood sheet fixed to the column top with bolts in order to provide dynamic excitation. The shaker was also suspended from the ceiling in the event the column collapsed.

The column was instrumented with 3 uniaxial accelerometers measuring accelerations in the direction of shaking (x -direction), perpendicular to shaking (y -direction) and the shaker armature. Instrumentation of the shaker armature allowed the input into the system to be measured. This was convenient for modal testing. Data was sampled at 200Hz using a computer fitted with a data acquisition card. Testing was conducted in the Civil Materials

Lab of the Department of Civil and Environmental Engineering, the University of Auckland
in June 2007.



(a)



(b)

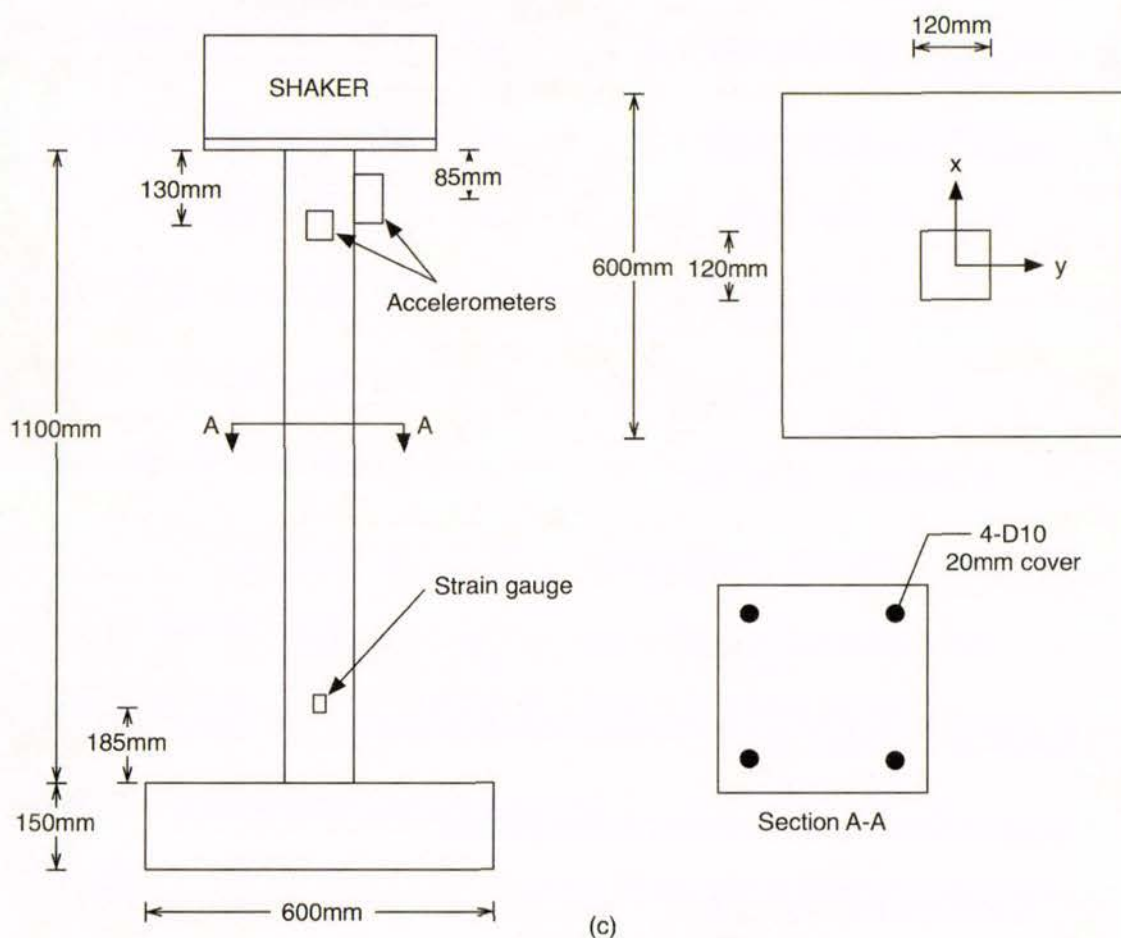


Figure 3.11. RC column: (a) general arrangement, (b) shaker, and (c) drawings.

3.2.1. Modal analysis

Modal analysis was conducted on the column using the Eigenvalue Realization Algorithm (ERA). Of primary interest was the translational mode in the x -direction, later used to estimate the lateral stiffness of the column in each damage state. Initially, a FRF was obtained for the x -direction using Welch's method with a Hanning window and no overlapping. This FRF showed two modes, a translational mode at 11.1Hz and a torsional mode at 9.0Hz, see Figure 3.12. Using the half-power bandwidth method, discussed above, modal damping was estimated to be 3.6% and 4.7% for the translational and torsional mode, respectively. The Inverse Discrete Fourier Transform was applied to the FRF and the system's Impulse Response Function was obtained. The IFR was required for ERA.

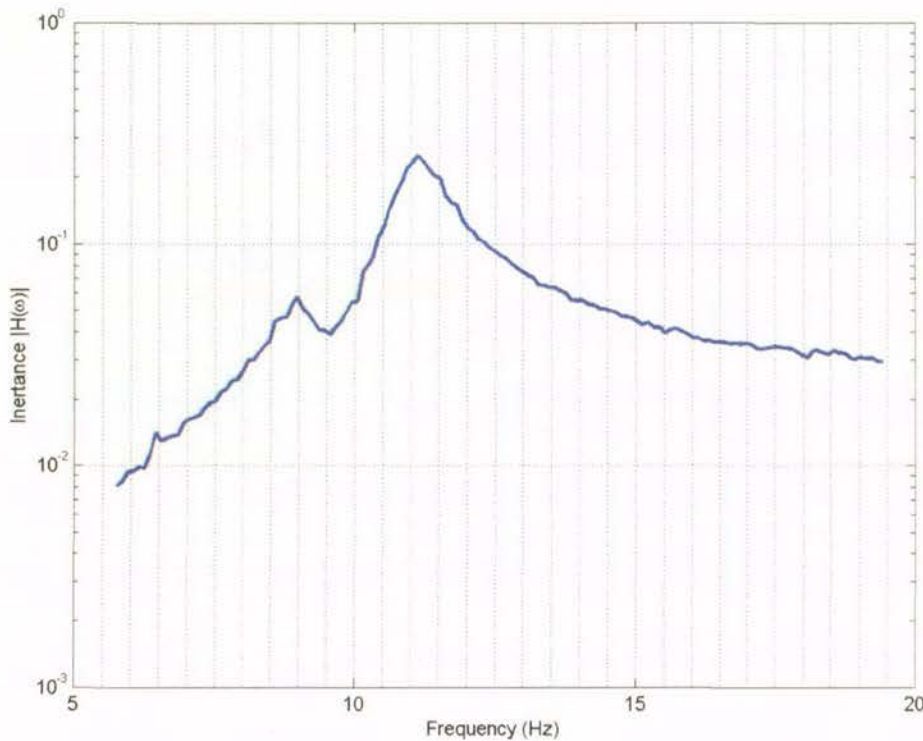


Figure 3.12. RC column FRF in x -direction.

ERA was developed by Juang (1994) and can be applied to both response only or ambient data, and forced vibration data where the input is known. The general algorithm is outlined below and follows the development in Juang (1994). In the identification algorithm, the state-space matrix triplet **A**, **B** and **C** will be estimated. Assuming the initial state of the state-space model in Equations (3.11) and (3.12) is zero, the impulse response of the system can be calculated from Equations (3.11) and (3.12) with the input $\mathbf{u}_0 = 1$ and $\mathbf{u}_k = 0$ for $k \neq 0$:

$$\mathbf{y}_0 = \mathbf{D}, \quad \mathbf{y}_1 = \mathbf{CB}, \quad \mathbf{y}_2 = \mathbf{CAB}, \quad \dots, \quad \mathbf{y}_k = \mathbf{CA}^{k-1}\mathbf{B} \quad (3.21)$$

where $\mathbf{y}_k$ are the Markov parameters or Impulse Response Functions (IRFs) of the system. The Markov parameters can be assembled to form a generalised $\alpha \times \beta$ block Hankel matrix $\mathbf{H}_{k-1}$ for time step k :

$$\mathbf{H}_{k-1} = \begin{bmatrix} \mathbf{y}_k & \mathbf{y}_{k+1} & \cdots & \mathbf{y}_{k+\beta-1} \\ \mathbf{y}_{k+1} & \mathbf{y}_{k+2} & \cdots & \mathbf{y}_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_{k+\alpha-1} & \mathbf{y}_{k+\alpha} & \cdots & \mathbf{y}_{k+\alpha+\beta-2} \end{bmatrix} \quad (3.22)$$

When $k = 1$, $\mathbf{H}_0$ is given by

$$\mathbf{H}_0 = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 & \cdots & \mathbf{y}_\beta \\ \mathbf{y}_2 & \mathbf{y}_3 & \cdots & \mathbf{y}_{1+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{y}_\alpha & \mathbf{y}_{1+\alpha} & \cdots & \mathbf{y}_{\alpha+\beta-1} \end{bmatrix} \quad (3.23)$$

If $\alpha \geq n$ and $\beta \geq n$, $\mathbf{H}_{k-1}$ is of rank n , where n is the order of the system. The $\mathbf{H}_0$ matrix can be factorised using singular value decomposition:

$$\mathbf{H}_0 = \mathbf{R}\mathbf{\Sigma}\mathbf{S}^T \quad (3.24)$$

where the columns of $\mathbf{R}$ and $\mathbf{S}$ are orthonormal and $\mathbf{\Sigma}$ is a rectangular matrix of the following form:

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (3.25)$$

with

$$\mathbf{\Sigma}_n = \text{diag}[\sigma_1, \sigma_2, \dots, \sigma_i, \dots, \sigma_n] \quad (3.26)$$

where σ_i ($i = 1, 2, \dots, n$) is a monotonically decreasing series of singular values. $\mathbf{H}_0$ can be constructed using only the non-zero singular values and the first n columns of the singular vectors $\mathbf{R}_n$ and $\mathbf{S}_n$:

$$\mathbf{H}_0 = \mathbf{R}_n \mathbf{\Sigma}_n \mathbf{S}_n^T \quad (3.27)$$

It can be shown (Juang 1994) that the minimum realization of the triplet is given by

$$\hat{\mathbf{A}} = \Sigma_n^{-1/2} \mathbf{R}_n^T \mathbf{H}_1 \mathbf{S}_n \Sigma_n^{-1/2} \quad (3.28)$$

$$\hat{\mathbf{B}} = \Sigma_n^{1/2} \mathbf{S}_n^T \mathbf{E}_r \quad (3.29)$$

$$\hat{\mathbf{C}} = \mathbf{E}_m^T \mathbf{R}_n \Sigma_n^{1/2} \quad (3.30)$$

where $\mathbf{E}_r = [\mathbf{I}_r \mathbf{0}_r \dots \mathbf{0}_r]^T$ and $\mathbf{E}_m = [\mathbf{I}_m \mathbf{0}_m \dots \mathbf{0}_m]^T$ are selection matrices with r and m equal to the number of inputs and measurements respectively.

ERA applied to the RC column data gave a translation mode of 11.2Hz with 4.1% damping. The torsional mode was estimated at 9.35Hz with 6.0% damping. Table 3.6 lists the frequency and damping ratio obtained using ERA and analysis of the FRF for both modes. The frequencies are in good agreement with a greater scatter associated with the damping ratios.

Table 3.6. Summary of RC column modal analysis results.

Mode	f (Hz)		ξ (%)	
	ERA	FRF	ERA	FRF
Translational	11.2	11.1	4.1	3.6
Torsional	9.35	9.0	6.0	4.7

3.3. The ASCE Phase II Experimental SHM Benchmark Structure

The ASCE Phase II Experimental SHM Benchmark Structure (ASCE Structural Health Monitoring Committee) is a 4-storey 2-bay by 2-bay steel frame with a 2.5m × 2.5m floor plan and a height of 3.6m, see Figure 3.13a. Note only a brief description of the structure and the test setup and programme is given herein. A full description can be found at the benchmark problem website (ASCE Structural Health Monitoring Committee). The columns were B100×9 sections and the floor beams were S75×11 sections, all sections were Grade 300 steel. The beams and columns were bolted together. Bracing was added in all bays with two 12.7mm diameter threaded steel rods, see Figure 3.13b. Additional mass was distributed around the structure to make it more realistic. Four 1000kg floor slabs were placed on the 1st, 2nd and 3rd floors, one per bay. On the 4th floor, four 750kg slabs were used. Two of the slabs per floor were placed off-centre to increase the coupling between translational and torsional motion.

A series of ambient and forced vibration tests were carried out on the structure at the University of British Columbia. In the following discussions the locations in the structure are referred to using their respective geographical directions of north (N), south (S), east (E) and west (W). Of primary interest in this study were the forced random vibration tests conducted using an electro-dynamic shaker mounted on the SW bay of the 4th floor on the diagonal. Input into the shaker was band-limited 5-50Hz white noise. The structure was instrumented with 15 accelerometers: 3 accelerometers each for the base, 1st, 2nd, 3rd and 4th floors. These were located on the E and W frames to measure motion in the N-S direction and in the centre to measure E-W motion. Acceleration data was recorded at 200Hz using a data acquisition system and filtered with anti-aliasing filters. In the case of this structure, modal analysis was not conducted and dynamic measurements were directly used for damage detection studies described later.



(a)



(b)

Figure 3.13. ASCE SHM Phase II Experimental Benchmark Structure: (a) general view, and (b) beam-column joint and bracing.

3.4. Conclusions

This chapter provided a description of the three experimental structures used later for damage detection studies. These structures are a 3-storey bookshelf structure, a RC column, and the ASCE Phase II Experimental SHM Benchmark Structure. The first two structures were specially built and tested for this project, while data from the third one was sourced from the

ASCE SHM Committee web site. The modal properties of the structures built and tested specifically for this study, i.e. the 3-storey bookshelf structure and the RC column, were determined from forced vibration tests. The ASCE Phase II Experimental SHM Benchmark Structure was tested elsewhere and its data was taken directly from that study.

The 3-storey bookshelf structure was excited using a shake table. A RC column was excited using a linear mass shaker mounted at the top of the column. The results showed that natural frequencies were the most accurately determined modal parameter obtained. Estimates for the 3-storey bookshelf structure using different system identification methods were between 1.87Hz-1.95Hz, 5.42Hz-5.52Hz and 8.43Hz-8.59Hz for the 1st, 2nd and 3rd modes, respectively. There was a greater uncertainty surrounding modal damping ratios. Observed damping in the 3-storey bookshelf structure was small and estimates of between 0.6%-1.6%, 0.3%-0.9% and 0.8%-1.5% critical damping were obtained for the 1st, 2nd, and 3rd mode, respectively. Natural frequencies of the RC column were estimated as 11.1-11.2Hz for the translational mode and 9.0-9.35Hz for the torsional mode. Damping in the RC column was much greater than in the 3-storey bookshelf structure and a better agreement between the different identification methods was obtained, 3.6%-4.1% for the translational mode and 4.7-6.0% for the torsional mode. Mode shapes obtained directly from acceleration data were in good agreement with those obtained from state-space identification for the 3-storey bookshelf structure. These experimental studies assisted in understanding the dynamics of the experimental structures and laid the ground for subsequent damage detection studies.

3.5. References

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CHAPTER 4

DAMAGE DETECTION AND QUANTIFICATION USING TIME SERIES ANALYSIS AND BACK PROPAGATION ARTIFICIAL NEURAL NETWORKS

In this chapter, a time-series based SHM method was developed and experimentally verified on three experimental structures described in Chapter 3: the 3-storey bookshelf structure, the RC column, and the ASCE Phase II Experimental SHM Benchmark Structure. The structures presented varying degrees of structural complexity from a simple cantilever, through a simple shear type structure to a multiple-storey, multiple-bay braced steel frame. Each structure was subjected to several simulated damage scenarios.

The proposed approach to damage detection was to fit the acceleration time histories obtained from the structure in undamaged and damaged states using AR models. The coefficients of these AR models were chosen as damage sensitive features and used as inputs into a BP ANN. The ANN was trained to recognise changes in the patterns of the AR coefficients and relate these changes to either a specific damage state (damage classification) or a reduction in structural stiffness (damage quantification). Structural stiffness in healthy and damaged states was identified from experimental data using model updating. For the ASCE Phase II Experimental SHM Benchmark Structure the number of AR coefficients was significant and two data reduction techniques were investigated. The feature dimension was reduced by either selecting a subset of the AR coefficients or projecting the AR coefficients using Principal Component Analysis (PCA). Significant reductions in computational burden were achieved whilst maintaining good accuracy of damage classification. The effect of changing operating conditions, simulated by a change in mass was investigated on the RC column

structure. Results showed good damage quantification was still obtainable under such conditions. In all three structures, ANNs were shown to be able to detect and classify or quantify damage with good accuracy.

The layout of this chapter is as follows. Firstly analytical techniques used are described. These include time series analysis, BP ANNs, model updating and PCA. Secondly, the proposed damage detection methodology is applied to each of the three experimental structures and results are discussed.

4.1. Time series analysis

Time series analysis techniques were originally developed for analysing long sequences of regularly sampled data and have been used in a wide range of fields where modelling or forecasting of a particular process is required. Due to the varied application of time series techniques to linear, nonlinear and seasonal data, a broad range of time series models have been developed. The selection of an appropriate model depends on both the statistical properties of the time series and modelling requirements.

A general structure of time series suitable for linear stationary processes is the Autoregressive-Moving Average with eXogenous input (ARMAX) model (Ljung 1999). A stationary process is a stochastic process in which the mean, variance and higher order moments are time invariant. A $ARMAX(n_a, n_b, n_c)$ model of Autoregressive (AR) order n_a , exogenous input of order n_b and Moving Average (MA) order n_c for the time series $\{y_t\}$ ($t = 1, 2, \dots, n$) with exogenous input $\{u_t\}$ ($t = 1, 2, \dots, n$) can be written as follows:

$$y_t + a_1 y_{t-1} + \dots + a_{n_a} y_{t-n_a} = b_1 u_{t-1} + \dots + b_{n_b} u_{t-n_b} + e_t + c_1 e_{t-1} + \dots + c_{n_c} e_{t-n_c} \quad (4.1)$$

where $a_1, \dots, a_{n_a}$, $b_1, \dots, b_{n_b}$ and $c_1, \dots, c_{n_c}$ are the AR, exogenous and MA coefficients, respectively, and $\{e_t\}$ is the residual error time series. The residual errors are assumed to be uncorrelated Gaussian white noise with zero mean and constant variance σ_e^2 . The model is parameterised by the vector θ which contains all the coefficients:

$$\theta = [a_1 \dots a_{n_a} \ b_1 \dots b_{n_b} \ c_1 \dots c_{n_c}]^T \quad (4.2)$$

By introducing the backshift operator B defined as

$$By_t = y_{t-1} \quad (4.3)$$

and the following polynomials in B :

$$A(B) = 1 + a_1B + \dots + a_{na}B^{na} \quad (4.4)$$

$$B(B) = b_1B + \dots + b_{nb}B^{nb} \quad (4.5)$$

$$C(B) = 1 + c_1B + \dots + c_{nc}B^{nc} \quad (4.6)$$

Equation (4.1) may be written in a more compact form:

$$A(B)y_t = B(B)u_t + C(B)e_t \quad (4.7)$$

By choosing different forms of the polynomials $A(B)$, $B(B)$ and $C(B)$ in Equation (4.7) a variety of different time series models can be obtained. Of primary interest in this research are AR models with or without the exogenous part.

4.1.1. Autoregressive Models

AR models are probably the simplest time series models available and used in the analysis of stationary time series. With the addition of the exogenous input, referred to as ARX models, a simple input-output relationship is obtained. AR models attempt to account for the correlations of the current observation in the time series with its predecessors. A short introduction to both AR and ARX models is given herein, a comprehensive discussion may be found in Box and Jenkins (1976) and Ljung (1999).

A univariate ARX model of order (na, nb) , or $ARX(na, nb)$, for the output time series $\{y_t\}$ can be written as

$$A(B)y_t = B(B)u_t + e_t \quad (4.8)$$

The ARX coefficients $a_1, \dots, a_{na}$ and $b_1, \dots, b_{nb}$ can be evaluated using a variety of methods (Ljung 1999). In this research, the coefficients were calculated using a least squares solution. Given a series of n observations, Equation (4.8) can be rewritten into matrix form:

$$\mathbf{e} = \mathbf{y} + \mathbf{Y}\boldsymbol{\theta} \quad (4.9)$$

where

$$\mathbf{e} = [e_n \dots e_{\max(na+1, nb+1)}]^T \quad (4.10)$$

$$\mathbf{y} = [y_n \dots y_{\max(na+1, nb+1)}]^T \quad (4.11)$$

$$\mathbf{Y} = \begin{bmatrix} y_{n-1} & \dots & y_{n-na} & -u_{n-1} & \dots & -u_{n-nb} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{\max(na+1, nb+1)-1} & \dots & y_{\max(na+1, nb+1)-na} & -u_{\max(na+1, nb+1)-1} & \dots & -u_{\max(na+1, nb+1)-nb} \end{bmatrix} \quad (4.12)$$

$$\boldsymbol{\theta} = [a_1 \dots a_{na} \ b_1 \dots b_{nb}]^T \quad (4.13)$$

In Equations (4.9)-(4.13), $\mathbf{e}$ is an error vector, $\mathbf{y}$ is a vector containing the current outputs, $\mathbf{Y}$ is a matrix of the previous outputs and inputs, and $\boldsymbol{\theta}$ is a vector containing the $na+nb$ ARX coefficients. A least squares solution seeks to minimise the sum of squared errors:

$$\mathbf{e}^T \mathbf{e} = (\mathbf{y} + \mathbf{Y}\boldsymbol{\theta})^T (\mathbf{y} + \mathbf{Y}\boldsymbol{\theta}) \quad (4.14)$$

Hence by differentiation of Equation (4.14) with respect to $\boldsymbol{\theta}$ the ARX coefficients can be calculated as

$$\boldsymbol{\theta} = (\mathbf{Y}^T \mathbf{Y})^{-1} \mathbf{Y}^T \mathbf{y} \quad (4.15)$$

4.2. Back-Propagation Artificial Neural Networks

In this section, the mathematical framework of BP ANNs is developed. ANNs are data processing structures consisting of several layers of interconnected neurons. While the neurons themselves are simple computational devices, collectively when interconnected the

result is a powerful parallel distributed processing system. The advantage of ANNs over other artificial intelligence systems, e.g. fuzzy logic, is that ANNs only require examples and no structured knowledge or IF – THEN rules. While there are many different types of ANNs as described in Chapter 2, for the purposes of the research presented in this Chapter the supervised learning technique of BP was of primary interest. Material presented in this section follows Kecman (2001).

ANNs utilising the error back-propagation algorithm are commonly referred to as BP ANNs and belong to a generalised class of multilayer perceptron networks. Prior to the development of the error back-propagation algorithm, networks were limited to a single output layer as there was no way the error and required weight change for the hidden layer could be calculated. With the addition of hidden layers the multilayer networks increased in computational power and were able to be used as universal approximators. The structure of BP networks comprise of interconnected layers of neurons, which are the basic computational units. Figure 4.1 shows structure of a single hidden layer network, where x_i are the I inputs into the network, y_k are the outputs of the K hidden layer neurons and o_j are the outputs of the J output layer neurons. The bias inputs, denoted by solid squares, into the hidden and output layers have indices of $I+1$ and $K+1$ respectively. Both biases have the value of +1. The hidden and output layer weights are denoted by w_{ik} and w_{kj} respectively, where $w_{\alpha\beta}$ is the weight for input α coming to neuron β .

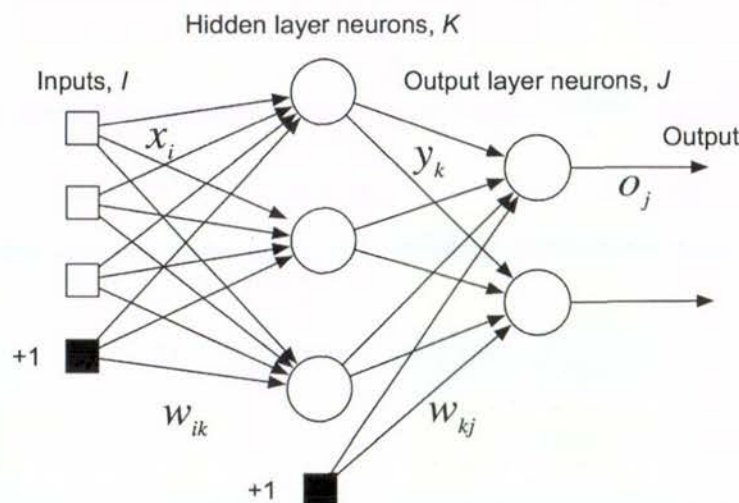


Figure 4.1. A single hidden layer BP ANN.

Figure 4.2 shows the function of an individual neuron, in this case a hidden layer neuron. The basic function of a neuron, in either the hidden or output layers, is to calculate the weighted sum of all inputs u_k and compute the output y_k . The weighted sum of all inputs is calculated using Equation (4.16). Note the notation below is for the k^{th} neuron in the hidden layer.

$$u_k = \sum_{i=1}^{I+1} x_i w_{ik} \quad (4.16)$$

The output of the neuron is computed using

$$y_k = f(u_k) \quad (4.17)$$

where f is the so-called neuron's activation function. Typical activation functions include tangent sigmoid, log-sigmoid, or linear. In this research the tangent sigmoid function was used.

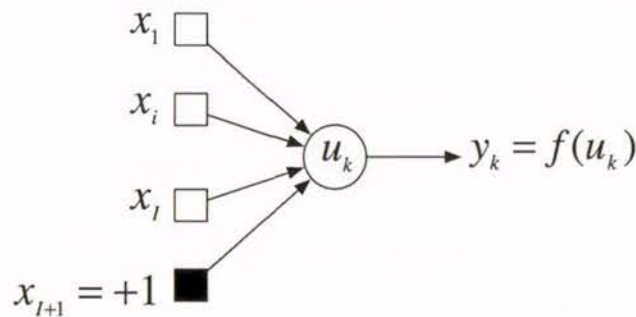


Figure 4.2. Function of the k^{th} neuron in the hidden layer.

Similarly, for the output layer neurons the sum of weighted inputs u_j and output o_j is given by Equations (4.18)-(4.19):

$$u_j = \sum_{k=1}^{K+1} y_k w_{kj} \quad (4.18)$$

$$o_j = f(u_j) \quad (4.19)$$

The error E_j at the j^{th} neuron in the output layer is defined by

$$E_j = \frac{1}{2} (d_j - o_j)^2 \quad (4.20)$$

where d_j is the target or desired value of the output j . The total error E in the network is therefore the sum of individual errors of Equation (4.20) over all J outputs:

$$E = \sum_{j=1}^J E_j = \frac{1}{2} \sum_{j=1}^J (d_j - o_j)^2 \quad (4.21)$$

In the error back-propagation algorithm, error is back propagated to the preceding layer. For Figure 4.1 the error at the output layer is propagated to the hidden layer. Following is a derivation of this algorithm for a single hidden layer network.

The goal is to minimise the error E by changing the weights w_{ik} and w_{kj} , using a gradient descent method (Rumelhart et al. 1986). Considering the error at the output layer the required change in weights Δw_{kj} is given by

$$\Delta w_{kj} = -\eta \frac{\partial E_j}{\partial w_{kj}}, \quad 0 < \eta < \eta_{crit} \quad (4.22)$$

where η is an unknown step size referred to as the learning rate, and is usually taken to be less than 1. If η is greater than η_{crit} the solution will not converge. Using the chain rule for differentiation Equation (4.22) can be expanded into

$$\Delta w_{kj} = -\eta \frac{\partial E_j}{\partial w_{kj}} = -\eta \frac{\partial E_j}{\partial o_j} \frac{\partial o_j}{\partial u_j} \frac{\partial u_j}{\partial w_{kj}} \quad (4.23)$$

The components of Equation (4.23) can be evaluated to give

$$\frac{\partial E_j}{\partial o_j} = -(d_j - o_j) \quad (4.24)$$

$$\frac{\partial o_j}{\partial u_j} = \frac{df(u_j)}{du_j} = f'(u_j) \quad (4.25)$$

$$\frac{\partial u_j}{\partial w_{kj}} = \frac{\partial \sum_{k=1}^{K+1} w_{kj} y_k}{\partial w_{kj}} = y_k \quad (4.26)$$

Equation (4.25) is simply the derivative of the activation function denoted by f' . Note this adds the constraint that the activation function must be differentiable if the error back-propagation algorithm is used. Substituting Equations (4.24)-(4.26) back into Equation (4.23) gives the required weight change for the output layer neurons:

$$\Delta w_{kj} = \eta (d_j - o_j) f'(u_j) y_k \quad (4.27)$$

The problem now is to find the error due to the hidden layer neurons. The total error from the output layer, E , is back propagated to the hidden layer and the error for the k^{th} hidden layer neuron becomes

$$E_k = E = \frac{1}{2} \sum_{j=1}^J (d_j - o_j)^2 \quad (4.28)$$

The starting point is again to use a gradient descent method to find the required weight change:

$$\Delta w_{ik} = -\eta \frac{\partial E_k}{\partial w_{ik}} \quad (4.29)$$

Using the chain rule for differentiation, Equation (4.29) can be expanded into

$$\frac{\partial E_k}{\partial w_{ik}} = \frac{\partial E_k}{\partial y_k} \frac{\partial y_k}{\partial w_{ik}} \quad (4.30)$$

The components of Equation (4.30) can be evaluated to give Equations (4.31)-(4.32):

$$\frac{\partial E_k}{\partial y_k} = \frac{\partial \sum_{j=1}^J E_j}{\partial y_k} = \sum_{j=1}^J \frac{\partial E_j}{\partial y_k} \quad (4.31)$$

$$\frac{\partial y_k}{\partial w_{ik}} = \frac{\partial y_k}{\partial u_k} \frac{\partial u_k}{\partial w_{ik}} \quad (4.32)$$

Applying the chain rule for differentiation to the derivative term in Equation (4.31) gives

$$\frac{\partial E_j}{\partial y_k} = \frac{\partial E_j}{\partial o_j} \frac{\partial o_j}{\partial u_j} \frac{\partial u_j}{\partial y_k} \quad (4.33)$$

The first two terms of Equation (4.33) have already been evaluated for the output layer, see Equations (4.24)-(4.25). The third term is given by

$$\frac{\partial u_j}{\partial y_k} = \frac{\partial \sum_{k=1}^{K+1} w_{kj} y_k}{\partial y_k} = w_{kj} \quad (4.34)$$

Finally, the terms in Equation (4.32) can be evaluated using Equations (4.35)-(4.36):

$$\frac{\partial y_k}{\partial u_k} = f'(u_k) \quad (4.35)$$

$$\frac{\partial u_k}{\partial w_{ik}} = x_i \quad (4.36)$$

Substituting the results back into Equation (4.29) gives the required weight changes for the hidden layer:

$$\Delta w_{ik} = \eta f'(u_k) x_i \sum_{j=1}^J [(d_j - o_j) f'(u_j) w_{kj}] \quad (4.37)$$

The two Equations (4.27) and (4.37) for the required weight changes for the output and hidden layers, respectively, can be simplified by introducing the concept of error signals. The error signals of the output δ_j and hidden δ_k layers are defined by

$$\delta_j = f'(u_j)(d_j - o_j) \quad (4.38)$$

$$\delta_k = f'(u_k) \sum_{j=1}^J \delta_j w_{kj} \quad (4.39)$$

Combining these error signals with the results of Equations (4.27) and (4.37) gives the following equations known as the Generalised Delta rule (Rumelhart et al. 1986):

$$\Delta w_{kj} = \eta \delta_j y_k \quad (4.40)$$

$$\Delta w_{ik} = \eta \delta_k x_i \quad (4.41)$$

Although the error back-propagation model was a major advance in the development of ANNs, the algorithm in its original form had two predominate problems. Firstly, it could easily get caught in a local minimum in error space and fail to find the global minimum. Secondly, it took a long time to converge to a result. To overcome these problems, several modified error back-propagation algorithms were developed.

The Levenberg-Marquardt algorithm (Marquardt 1963), a quasi-Newton method, was developed specifically for the sum of errors squared error function. The algorithm's application to ANNs is described in Hagan and Menhaj (1994). In matrix form, the error $E(\mathbf{w})$ is a function of the weights and can be written as

$$E(\mathbf{w}) = \mathbf{e}(\mathbf{w})^T \mathbf{e}(\mathbf{w}) \quad (4.42)$$

where $\mathbf{e}(\mathbf{w})$ is an error vector defined by

$$\mathbf{e}(\mathbf{w}) = \mathbf{d} - \mathbf{o}(\mathbf{w}) \quad (4.43)$$

The target value and output vectors are denoted as $\mathbf{d}$ and $\mathbf{o}(\mathbf{w})$ and defined as

$$\mathbf{d} = [d_1 \dots d_j \dots d_J]^T \quad (4.44)$$

$$\mathbf{o} = [o_1 \dots o_j \dots o_J]^T \quad (4.45)$$

The weights vector $\mathbf{w}$ is

$$\mathbf{w} = [w_{11} \ w_{12} \dots w_{ij} \dots w_{I+1 \ K} \ w_{11} \ w_{12} \dots w_{kj} \dots w_{K+1 \ J}]^T \quad (4.46)$$

Introducing the Jacobian matrix $\mathbf{J}$ defined by

$$\mathbf{J}(\mathbf{w}) = \frac{\partial \mathbf{e}}{\partial \mathbf{w}} \quad (4.47)$$

the new weights can be computed by the following iterative formula:

$$\mathbf{w}_{k+1} = \mathbf{w}_k - [\mathbf{J}(\mathbf{w}_k)^T \mathbf{J}(\mathbf{w}_k) + \lambda_k \mathbf{I}]^{-1} \mathbf{J}^T(\mathbf{w}_k) \mathbf{e}(\mathbf{w}_k) \quad (4.48)$$

where subscript k denotes the iteration step. The parameter λ_k is a scalar that controls convergence properties. If λ_k is equal to zero the Levenberg-Marquardt algorithm becomes the Gauss-Newton method. In this study all BP networks were trained using the Levenberg-Marquardt algorithm with an early-stopping criterion on validation data (Demuth et al. 2006). This prevented the network from overfitting the training data. Overfitting occurs when noise present in the data is modelled rather the underlying function causing a loss of generalisation abilities.

4.3. Model updating

In order to match experimental results, analytical models can be updated on a range of experimental data including natural frequencies, mode shapes and frequency domain data

(Friswell and Mottershead 1995). Natural frequencies are often the most certain modal parameters obtained from modal analysis and are often used in the model updating process, as is also the case in this study. The relative errors $\mathbf{e}_i$ between the analytical and experimental frequencies, $\omega_{a,i}$ and $\omega_{e,i}$, can be expressed as an error vector:

$$\mathbf{e}_i = \frac{\omega_{a,i}^2 - \omega_{e,i}^2}{\omega_{e,i}^2} \quad (4.49)$$

where subscript i refers to i^{th} mode. The analytical frequencies depend on a set of parameters, such as masses and member stiffnesses, which define the model and are denoted by vector $\boldsymbol{\theta}$. In the sensitivity-based updating process, the vector of errors for all considered modes $\mathbf{e}$ is minimized by an iterative procedure where the updating parameters $\boldsymbol{\theta}_k$ at iteration step k are adjusted using

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \mathbf{S}_k^+ \mathbf{e}_k \quad (4.50)$$

where $\mathbf{S}^+$ is the pseudoinverse of the sensitivity matrix whose entries can be evaluated as

$$S_{ij} = \frac{1}{\omega_{e,i}^2} \phi_{a,i}^T \left[\frac{\partial \mathbf{K}}{\partial \theta_j} - \omega_{a,i}^2 \frac{\partial \mathbf{M}}{\partial \theta_j} \right] \phi_{a,i} \quad (4.51)$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices and $\phi_{a,i}$ are the analytical mode shape vectors.

After the updating process is complete, the analytical mode shapes can be checked against the experimental mode shapes using the popular Modal Assurance Criterion (MAC) (Friswell and Mottershead 1995):

$$MAC_{ij} = \frac{|\phi_{a,i}^T \phi_{e,j}|^2}{(\phi_{a,i}^T \phi_{a,i})(\phi_{e,j}^T \phi_{e,j})} \quad (4.52)$$

where ϕ_a and ϕ_e are the analytical and experimental mode shapes. MAC values of 0.9 or greater are generally considered to be a sign of good correlation between analytical and experimental mode shapes.

4.4. Principal Component Analysis

Principal Component Analysis (PCA) is a well-known multivariate statistical technique and a full description can be found, e.g., in Sharma (1997). Given a set of n data points $\mathbf{x}_i = [x_{1i}, x_{2i},$

$\dots, x_{pi}]^T$ ($i = 1, \dots, n$) in a p -dimensional space with mean $\bar{\mathbf{x}}$ and covariance matrix Σ , PCA seeks to project the data into a new p -space with orthogonal coordinates $\mathbf{z}_i$ ($i = 1, \dots, n$) via a linear transformation.

Decomposition of the covariance matrix by singular value decomposition leads to

$$\Sigma = \mathbf{V}\Lambda\mathbf{V}^T \quad (4.53)$$

where $\Lambda = \text{diag}[\sigma_1^2, \dots, \sigma_p^2]$ is diagonal matrix containing the ranked eigenvalues of Σ and $\mathbf{V}$ is a matrix containing the corresponding eigenvectors or principal components. The transformation of a data point $\mathbf{x}_i$ into principal components is

$$\mathbf{z}_i = \mathbf{V}^T (\mathbf{x}_i - \bar{\mathbf{x}}) \quad (4.54)$$

The new coordinates $\mathbf{z}_i$ are uncorrelated and have a diagonal covariance matrix Λ . Therefore, $\mathbf{z}_i = [z_{1i}, \dots, z_{pi}]^T$ is a linear combination of $x_{1i}, \dots, x_{pi}$ which explain variances $\sigma_1^2, \dots, \sigma_p^2$. To reduce the dimensionality, a selection $q < p$ of principal components can be used that retains those components that contribute most to the variance, thus reducing the dimension of the data to q .

4.5. Application to 3-storey bookshelf structure

In this section, the proposed damage detection method using BP ANNs was applied to the 3-storey bookshelf structure described in Chapter 3. Damage was introduced into the structure by replacing the original 4.5mm thick columns of a particular storey with thinner, 3.0mm aluminium angles. Four damage states were considered; these were labelled D0, D1, D2 and D3 corresponding to no damage (healthy structure), 1st storey damage, 2nd storey damage and simultaneous 1st and 2nd storey damage.

Before damage classification and quantification the modal properties of the structure in either undamaged or damaged states were obtained. This data was used to update simple mass-spring models of the structure. Modal parameters when the structure was in both undamaged and damaged states were estimated from five response records, containing four 10s and one 20s record using the procedures explained in Chapter 3. In all cases the ground excitation was chosen to be Gaussian white noise. Table 4.1 shows the estimated natural frequencies f_e and

percentage changes of the frequencies $\Delta f_e/f_e$ in states D1, D2, and D3 in relation to D0. Mode shapes normalised for a maximum response of 1 are shown in Table 4.2 and graphically in Figure 4.1 for all four damage states.

Table 4.1. Natural frequencies and their percentage changes at different damage states for 3-storey bookshelf structure.

Mode	f_e (Hz)				$\Delta f_e/f_e$ (%) ^a		
	D0	D1	D2	D3	D1	D2	D3
1 st	1.928	1.879	1.837	1.840	-2.5%	-4.7%	-4.6%
2 nd	5.52	5.43	5.46	5.42	-1.6%	-1.0%	-1.8%
3 rd	8.55	8.30	8.09	8.15	-2.9%	-5.3%	-4.6%

^aBased on D0.

Table 4.2. Mode shapes at different damage states for 3-storey bookshelf structure.

Storey	Mode shape 1				Mode shape 2				Mode shape 3			
	D0	D1	D2	D3	D0	D1	D2	D3	D0	D1	D2	D3
1 st	0.18	0.20	0.10	0.20	0.64	0.69	0.68	0.54	1.00	1.00	1.00	1.00
2 nd	0.69	0.70	0.66	0.71	1.00	1.00	1.00	1.00	-0.48	-0.53	-0.52	-0.53
3 rd	1.00	1.00	1.00	1.00	-0.88	-0.91	-0.87	-0.86	0.16	0.18	0.19	0.19

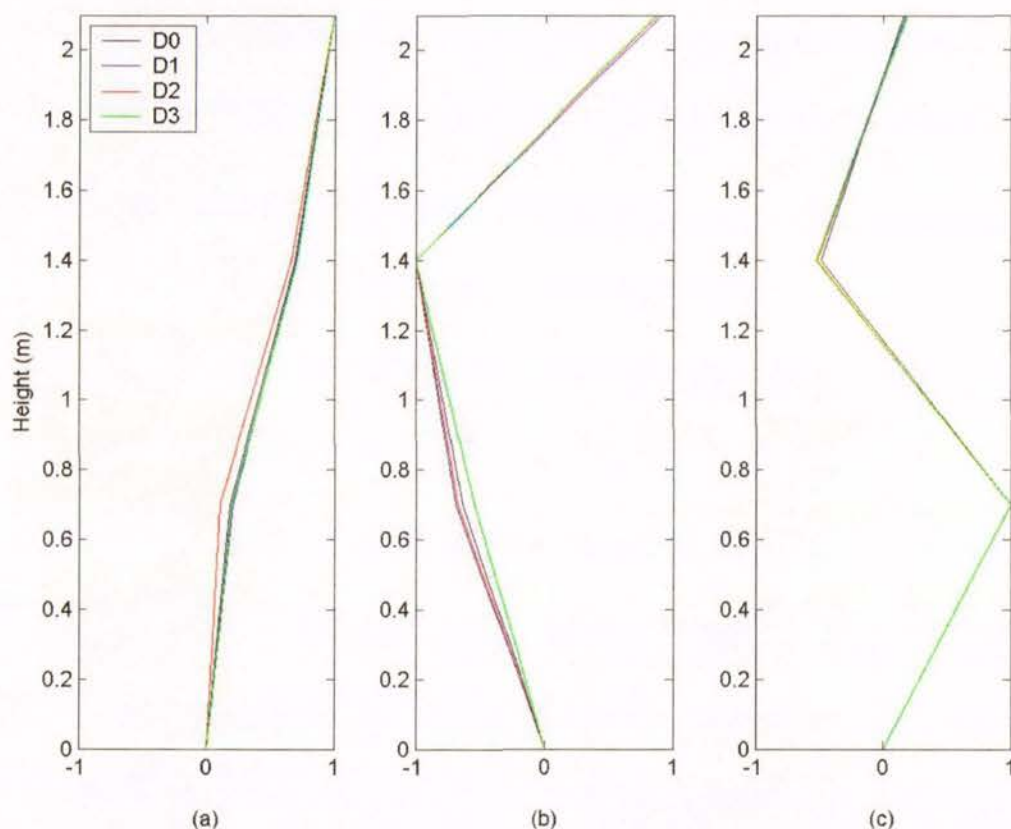


Figure 4.3. Mode shapes of 3-storey bookshelf structure in each damage state: (a) 1st mode, (b) 2nd mode, and (c) 3rd mode.

Eight scaled earthquake records were used to excite the structure in the four damage states. Table 4.3 lists the earthquakes used, the Peak Ground Acceleration (PGA) of the original and

scaled records, the duration of the record and the frequency at which the earthquake was sampled. The earthquakes were scaled so that a range of response amplitudes was obtained, while ensuring no yielding of the structure occurred. Yielding of the structure would cause nonlinearities in the structural response and such cases are outside the scope of this chapter.

Table 4.3. Earthquake records used to excite 3-storey bookshelf structure.

Earthquake	PGA (g)	Scaled PGA (g)	Duration (sec)	Sampling frequency (Hz)
Duzce 12/11/1999	0.535	0.027	25.885	200
Erzincan 13/3/1992	0.496	0.033	20.780	200
Gazli 17/5/1976	0.718	0.048	16.265	200
Helena 31/10/1935	0.173	0.035	40.000	100
Imperial Valley 19/5/1940	0.313	0.031	40.000	100
Kobe 17/1/1995	0.345	0.035	40.960	100
Loma Prieta 18/10/1989	0.472	0.047	39.945	200
Northridge 17/1/1994	0.568	0.038	40.000	50

4.5.1. Model updating

In this study, the lateral stiffness of the structure in both undamaged and damaged states could not be accurately determined from analytical investigations. The actual stiffness was much less than initially expected. This was attributed to the construction of the column-floor joints. The joints were constructed using relatively flexible brackets and lacked continuity of the column over the joint, refer to Figure 3.1c in Chapter 3. Knowledge of the stiffness would however be useful and necessary if experimental damage was to be quantified as a reduction in lateral stiffness. Also in a real-world application a computer model would have to be relied upon as it is highly unlikely that a full-scale structure would be damaged to obtain AR coefficients. Although outside the scope of this study, an accurate computer model of the structure would allow AR coefficients to be calculated at various simulated damage scenarios.

In order to estimate the stiffness, simple 3-DOF lumped mass-spring analytical models (Figure 4.4) were updated. The updating parameters were chosen to be the lateral stiffness of each storey k_1 , k_2 , and k_3 . The initial stiffnesses were estimated by hand calculations giving 9300N/m for k_2 and k_3 . Because of the additional brackets placed at the base of the structure, k_1 was expected to be significantly greater than k_2 or k_3 . As a hand calculation was difficult, a value for k_1 of 40000N/m was used.

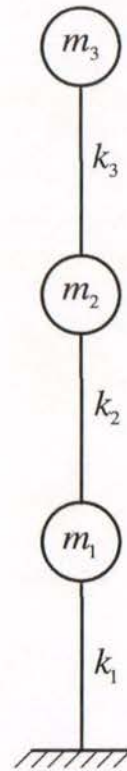


Figure 4.4. 3-DOF lumped mass-spring model used for modelling the 3-storey bookshelf structure.

Table 4.4 lists the results from the updated analytical models. The analytical frequencies f_a are very close to the experimental values. The MAC values show excellent correlation for the 1st mode while the 2nd and 3rd modes show good correlation. The obtained storey stiffnesses are given in Table 4.5.

Table 4.4. Analytical frequencies and MAC values after updating for 3-storey bookshelf structure.

	D0		D1		D2		D3	
Mode	f_a (Hz)	MAC ^a	f_a (Hz)	MAC ^a	f_a (Hz)	MAC ^a	f_a (Hz)	MAC ^a
1 st	1.928	1.00	1.879	0.99	1.837	1.00	1.840	0.99
2 nd	5.517	0.93	5.426	0.92	5.464	0.93	5.417	0.96
3 rd	8.548	0.93	8.303	0.92	8.093	0.92	8.152	0.91

^aBased on experimental mode shapes.

Table 4.5. Updated stiffnesses from analytical models for 3-storey bookshelf structure.

	Initial	D0	D1	D2	D3
k_1 (N/m)	4.00×10^4	3.77×10^4	3.55×10^4	3.38×10^4	3.43×10^4
k_2 (N/m)	0.93×10^4	0.61×10^4	0.58×10^4	0.54×10^4	0.54×10^4
k_3 (N/m)	0.93×10^4	0.78×10^4	0.76×10^4	0.78×10^4	0.76×10^4

From the updated model stiffnesses, the damaged and undamaged stiffness for each storey was estimated by averaging the results in Table 4.5. As no changes were made at the 3rd

storey, k_3 should remain constant for all damage states and therefore an average of all values in Table 4.4 was taken. Similarly, the undamaged k_2 should remain constant for states D0 and D1, while the damaged k_2 should be constant for states D2 and D3. The process was repeated for k_1 , however, k_1 for D2 was considered to be too low and was ignored. The final, averaged stiffnesses are given in Table 4.6. This averaging of the stiffness affected the analytical frequencies and MAC values and the final values have been listed in Table 4.7.

Table 4.6. Averaged stiffness for damage states for 3-storey bookshelf structure.

	D0	D1	D2	D3
k_1 (N/m)	3.77×10^4	3.49×10^4	3.77×10^4	3.49×10^4
k_2 (N/m)	0.60×10^4	0.60×10^4	0.54×10^4	0.54×10^4
k_3 (N/m)	0.77×10^4	0.77×10^4	0.77×10^4	0.77×10^4

Table 4.7. Final analytical frequencies and MAC values for 3-storey bookshelf structure based on averaged model updating results.

	D0		D1		D2		D3	
Mode	f_a (Hz)	MAC ^a	f_a (Hz)	MAC ^a	f_a (Hz)	MAC ^a	f_a (Hz)	MAC ^a
1 st	1.917	1.00	1.908	0.99	1.846	1.00	1.838	0.99
2 nd	5.489	0.93	5.473	0.93	5.458	0.91	5.445	0.96
3 rd	8.541	0.93	8.283	0.93	8.462	0.90	8.200	0.91

^aBased on experimental mode shapes.

4.5.2 Damage classification using output-only model

The acceleration time history of each storey was modelled using a univariate AR model. The same data acquisition procedure applied in Chapter 3 was followed. However, for the time series modelling of the acceleration data the signal was decimated from 400Hz to 50Hz. A univariate AR(12) model was determined to give both a sufficient fit to the acceleration data and had no significant correlation in the residual errors. The AR coefficients were estimated from a 500-point window advancing 100 points until the end of the record was reached. A least squares approach was used to calculate the AR coefficients. A data set of 388 points containing 97 points for each damage state was obtained. Figure 4.5 shows the statistical distribution (histogram) of the 1st AR coefficient from the 1st storey in the D0 and D1 damage states. There are noticeable changes in spread and mean between the two distributions which both appear to be bi-modal. However, extracting further information e.g., the location or extent of damage however requires more investigation.

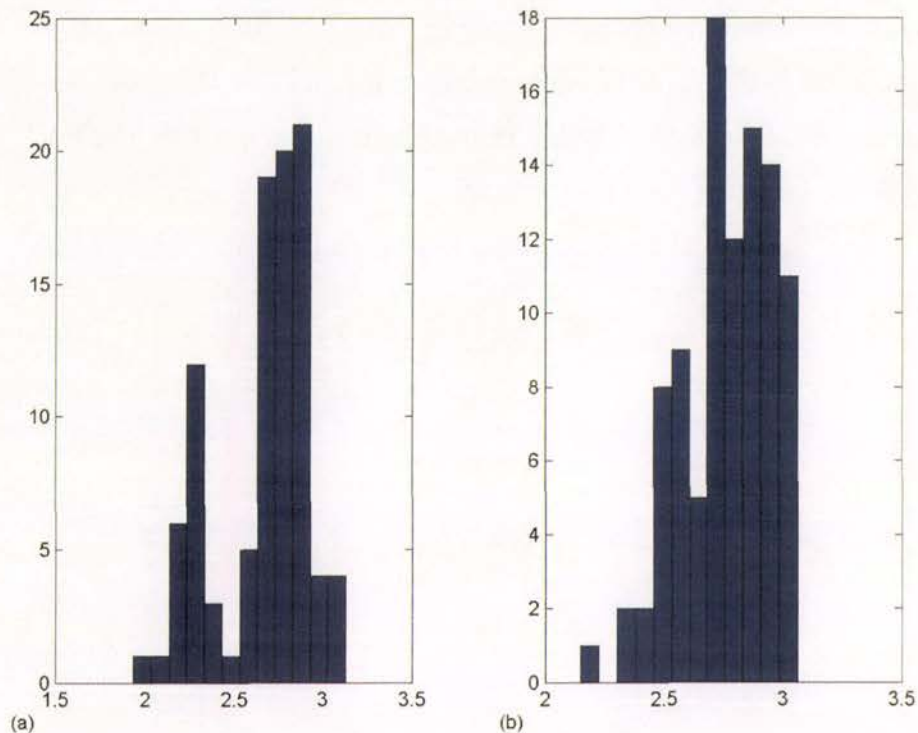


Figure 4.5. Histograms of the 1st AR coefficient for 3-storey bookshelf structure in various damage states: (a) D0, and (b) D1.

The 388 point data set was randomly divided into 300 points for training and 88 points for testing the ANNs. Initially, the ANN was trained to distinguish between the four damage states only. The damage states D0, D1, D2 and D3 were assigned the vector outputs $[1 \ 0 \ 0 \ 0]^T$, $[0 \ 1 \ 0 \ 0]^T$, $[0 \ 0 \ 1 \ 0]^T$ and $[0 \ 0 \ 0 \ 1]^T$, respectively. A single hidden layer ANN with 5 hidden layer neurons was found to give perfect results with 100% correct classification. Table 4.8 shows the break down of the classification results.

Table 4.8. Damage classification results for 3-storey bookshelf structure.

	D0	D1	D2	D3
Actual number of data points	27	17	26	18
Classified correctly	27	17	26	18

4.5.3 Damage detection, localisation and quantification using output-only model

Rather than simply classifying damage into several classes, a more useful approach would give information about the extent and location of damage in the structure. In this section, the ANNs were trained to relate the AR coefficients to the remaining stiffness at each storey, providing more useful information about the extent and location of damage. The damage at each storey is listed in Table 4.9 and shown in Figure 4.6 as the percentage of remaining storey lateral stiffness. A single hidden layer ANN with 5 hidden layer neurons was found to

give good predictions. The results have been shown graphically in Figure 4.7, where the detected damage has been plotted against the actual damage for all three stories. For perfect predictions, the data points should lie on (0.93,0.93) and (1.00,1.00) for the 1st storey, (0.90,0.90) and (1.00,1.00) for the 2nd storey and (1.00,1.00) for the 3rd storey. The means of the identified remaining stiffness values together with two standard deviation bounds are shown in Table 4.10. At a 95% confidence level these values do not differ from the actual stiffness identified by model updating by more than 3.3%. These results show that the ANN has correctly quantified the damage at each story with only a small amount of scatter about the actual remaining stiffness.

Table 4.9. Damage at each storey as a percentage of remaining stiffness for 3-storey bookshelf structure.

Storey	Percentage of remaining stiffness (%)			
	D0	D1	D2	D3
1 st	1.00	0.93	1.00	0.93
2 nd	1.00	1.00	0.90	0.90
3 rd	1.00	1.00	1.00	1.00

Table 4.10. ANN identified damage as a percentage of remaining stiffness in 3-storey bookshelf structure using AR models.

Storey	Identified percentage of remaining stiffness	
	Undamaged	Damaged
1 st	1.00±0.02	0.94±0.02
2 nd	1.00±0.02	0.90±0.03
3 rd	1.00±0.00	-

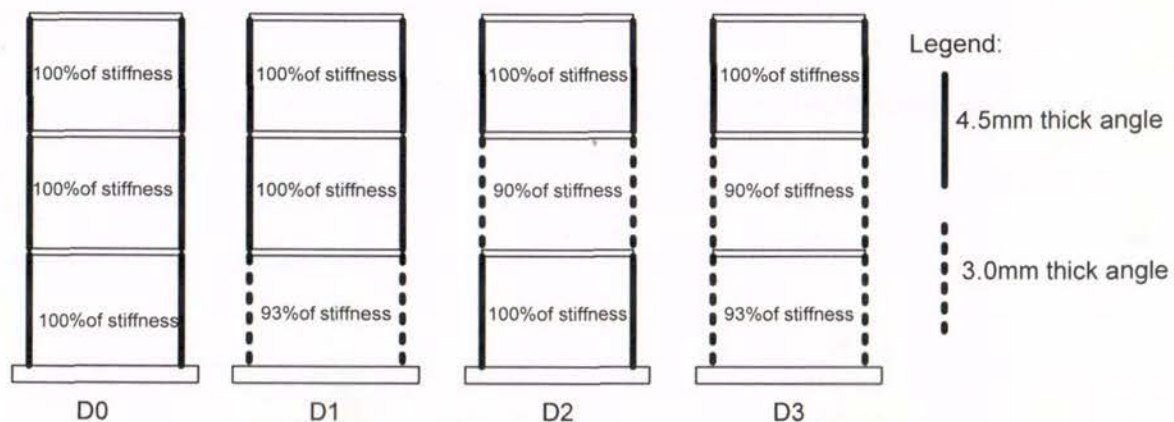


Figure 4.6. Damage states in 3-storey bookshelf structure showing percentage of lateral stiffness at each story.

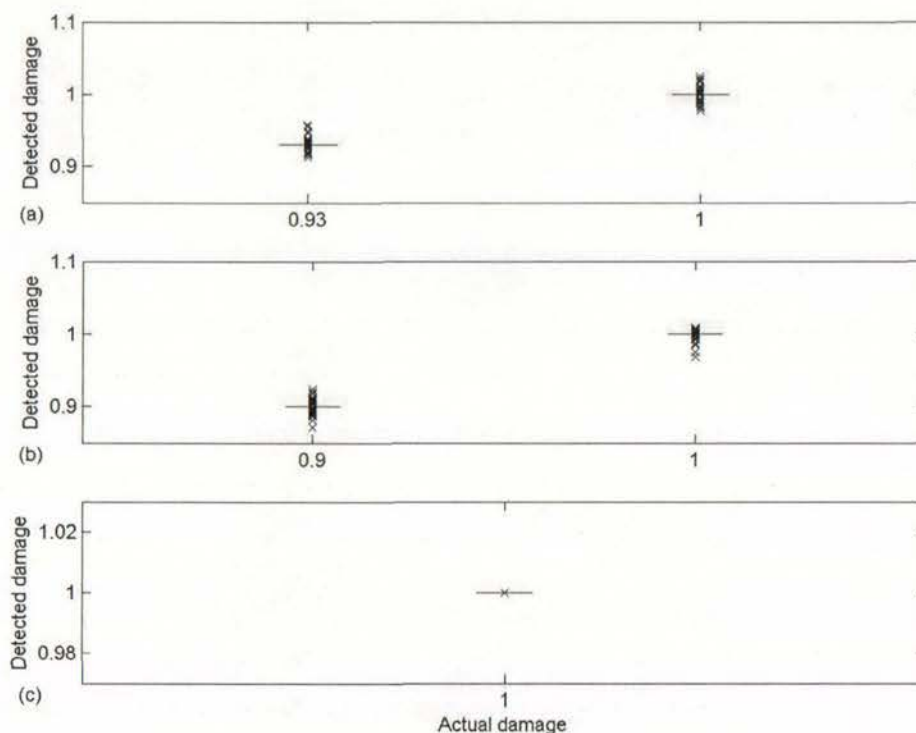


Figure 4.7. Detected vs. actual damage for 3-storey bookshelf structure using AR models: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

4.5.4 Damage detection, localisation and quantisation with a known input

In this study it is assumed that additional information is available, i.e. the input into the system as measured by the accelerometer fitted to the shake table. Inclusion of the input into the time series model could improve accuracy and can be achieved using the ARX formulation, a simple input-output model. An ARX(12,12) model was selected and applied to the time histories of each storey obtained above. The ARX coefficients were estimated using least squares from a 500-point window advancing 100 points until the end of the record was reached. A data set of 388 points containing 97 points for each damage state was obtained and randomly divided into 300 and 88 points for training and testing respectively. The results have been shown in Figure 4.8. Compared to Figure 4.7 there is a clear reduction in the amount of scatter about the actual value of remaining storey stiffness. Table 4.11 shows the means and two standard deviation bounds for the percentage remaining stiffness identified by the ANN using the ARX models. At a 95% confidence level these values do not differ from the actual stiffness identified by model updating by more than 1.1%. The means the error bounds have been reduced by a factor of 2 or 3 from those in Table 4.10.

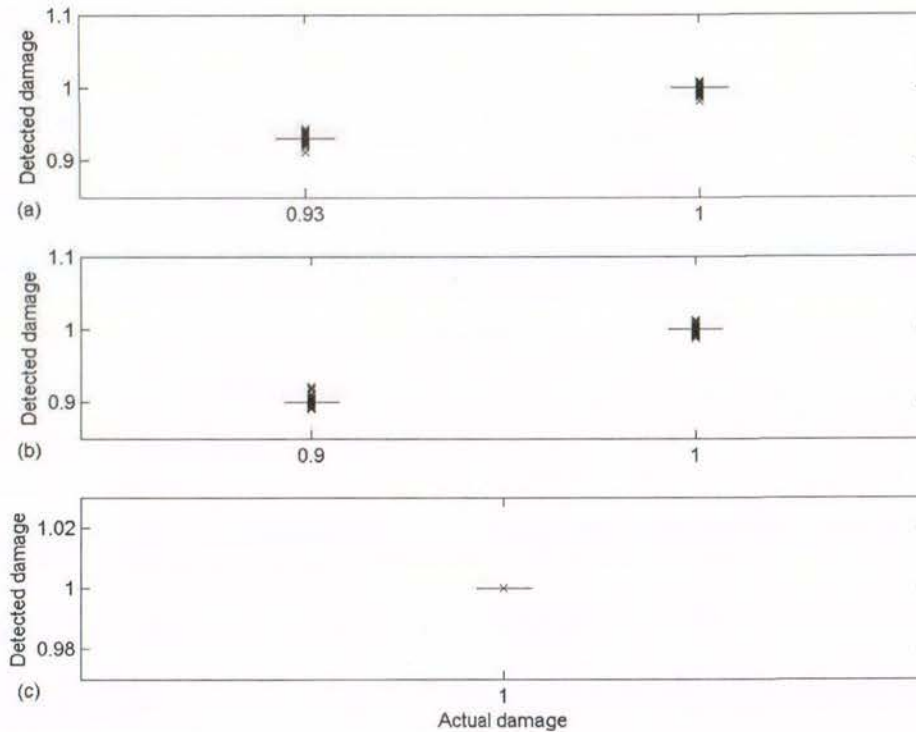


Figure 4.8. Detected vs. actual damage for 3-storey bookshelf structure using ARX models: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

Table 4.11. ANN identified damage as a percentage of remaining stiffness in 3-storey bookshelf structure using ARX model.

Storey	Identified percentage of remaining stiffness	
	Undamaged	Damaged
1 st	1.00±0.01	0.93±0.01
2 nd	1.00±0.01	0.90±0.01
3 rd	1.00±0.00	-

4.6. Application to ASCE Phase II Experimental SHM Benchmark Structure

A basic description of the ASCE Phase II Experimental SHM Benchmark Structure and experimental programme is provided in Chapter 3. In the following discussions the locations in the structure are referred to using their respective geographical directions of north (N), south (S), east (E) and west (W). A total of 9 damage scenarios were simulated on the structure; these involved the removal of bracing and the loosening of bolts in the floor beam connections. Table 4.12 lists the damage states and gives a description of damage. The different configurations give a mixture of minor and extensive damage cases.

Table 4.12. Damage configurations for ASCE Phase II Experimental Benchmark Structure.

Configuration	Damage
1	No damage
2	All bracing removed on the E side
3	Bracing removed floors 1-4 on a bay on the SE corner
4	Bracing removed floors 1 and 4 on a bay on the SE corner
5	Bracing removed floors 1 on a bay on the SE corner
6	All bracing removed on E face and floor 2 on N face
7	All bracing removed
8	Configuration 7 + loosened bolts on floors 1-4 on E face N bay
9	Configuration 7 + loosened bolts on floors 1 and 2 on E face N bay

4.6.1. Damage classification

The data from the ASCE Phase II Experimental SHM Benchmark Structure served a twofold purpose. Firstly, it was used to validate the performance of the proposed method on a more realistic and complex structure with complex damage scenarios. Secondly, the problem of data reduction for multi-sensor SHM systems proved to be of importance and was addressed.

Because quantitative information about the damage severity was not readily available, in this application damage classification into the 9 states was attempted. The 9 damage configurations were assigned vector outputs from $[1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$ to $[0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T$ for configurations 1 to 9, respectively. Univariate AR(20) models were fitted to the acceleration data from each accelerometer. The AR coefficients were estimated using least squares from 1000-point segments advancing 200 points until the end of the record was reached. A data set of 1035 points was obtained, 115 points from each configuration, and randomly divided into 700 points for training and 335 points for testing, respectively.

Using AR(20) models and all 15 accelerometers gave an ANN input dimension of 300. This sized network proved to be difficult to train due to computational limitations. Two data reduction techniques were investigated: (i) selection of subset AR coefficients and/or accelerometers and (ii) projection of the data onto a lower dimensional space using PCA.

Reducing the number of AR coefficients and/or accelerometers may adversely affect the amount of information in the data and degrade the performance of the damage detection method. Therefore several combinations of reduced AR coefficients and accelerometers were systematically investigated to ascertain their practical minimum numbers. The number of AR coefficients was reduced by selecting the first few coefficients only. This was chosen to be either (i) the first coefficient, (ii) the first two coefficients, (iii) the first three coefficients, (iv)

the first four coefficients, or (v) the first six coefficients. Similarly, the number of accelerometers and their location was either (i) the full set (15 accelerometers), (ii) omitted accelerometers located on the base and those of the W face measuring N-S motion (8 accelerometers), or (iii) same as case (ii) but with all remaining accelerometers on stories 1 and 3 omitted (4 accelerometers). Choosing n_{accel} accelerometers and n_{AR} AR coefficients reduces the damage sensitive feature dimension to $n_{accel} \times n_{AR}$ and this can be compared to the original dimension of 300. A single hidden layer ANN with 3 hidden layer neurons was selected for its good performance. The results were analysed as a classification task and are shown in Figure 4.9 where the number of misclassifications out of the 335-point test data set is given for different combinations of reduced AR coefficients and/or accelerometers. Establishing the value of a threshold where the performance of an SHM system can be judged as good or otherwise is always rather arbitrary. Here, 5% or less of misclassifications was considered to be a good result and 1% or less to be excellent. The quality of results is referred to using the same words throughout the whole report. Figure 4.9 shows a clear boundary where performance rapidly deteriorates below the 5% misclassification threshold. The full suite of 15 accelerometers achieved good performance with only 1 AR coefficient (reduced feature dimension 15), while 2 or 3 AR coefficient were required for 8 or 4 accelerometers, respectively, (reduced feature dimension 16 or 12, respectively). For excellent results at least 2 or 3 AR coefficients were necessary for 15 and 8 accelerometers, respectively, (reduced feature dimension 30 or 24, respectively). An overall conclusion that can be drawn from this simulation is that small numbers of AR coefficients and sensors suffice for precise damage classification. Compared to the original feature dimension of 300, depending on the choice of accelerometers only about 5% of features were required to obtain good or excellent results.

While the dimensionality reduction approach discussed above gave good results, a more methodical approach would retain data of statistical significance only. PCA was used to project the data onto the first 30, 20, 10, or 5 principal components. These, compared to the original damage sensitive feature dimension, correspond to 10%, 6.7%, 3.3% and 1.7% of this original dimension. Using a single hidden layer ANN with 5 hidden layer neurons, the number of classifications with percentage errors for each case is given in Table 4.13. The results showed good damage classification could be achieved by projection of the data onto the first 10 or more principal components. Some improvements over the previous results were observed especially when the feature dimension was small. Referring to Figure 4.9, using 4 accelerometers and 3 AR coefficients, equating to a feature dimension of 12, 46

misclassifications were recorded. When using 10 principal components, 16 misclassifications were recorded. This comparison shows a superior performance of the more systematic feature reduction approach using PCA over simple selection of accelerometers and AR coefficients. Excellent or perfect classification results were obtained when using 20 or more principal components.

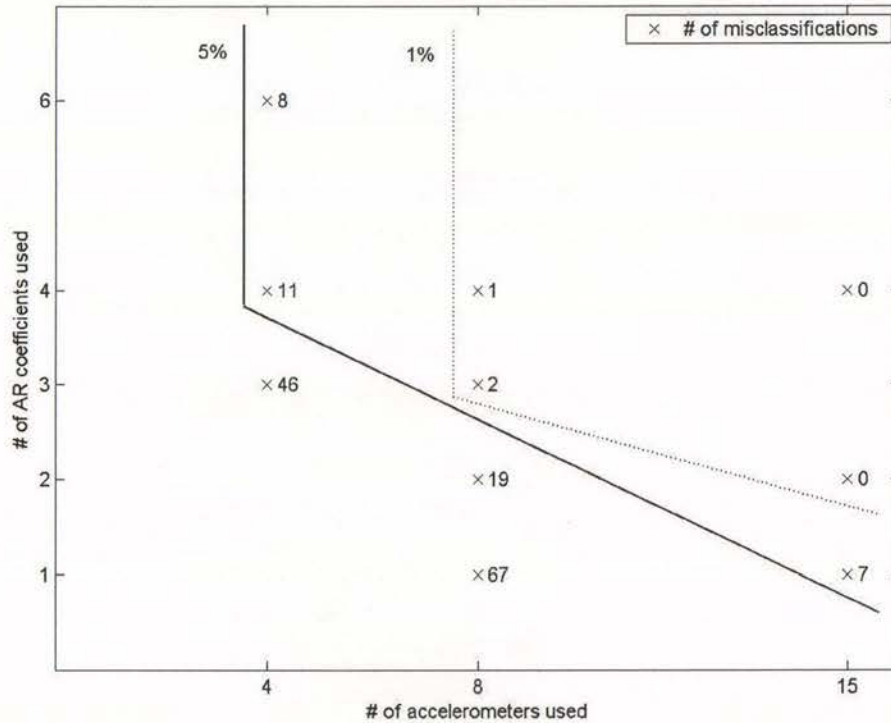


Figure 4.9. Results from the ASCE Phase II Experimental SHM Benchmark Structure using subsets of AR coefficients and accelerometers.

Table 4.13. Number of misclassifications using PCA reduced data from the ASCE Phase II Experimental SHM Benchmark Structure.

Number of principal components	Number of misclassifications
30	0 (0%)
20	2 (0.6%)
10	16 (4.8%)
5	85 (25.4%)

4.7. Application to the RC column

The RC column described in Chapter 3 was damaged progressively from minor to severe damage cases by chipping off concrete from around the base using a chisel and hammer. This method of introducing damage was adopted because the linear shaker was unable to provide sufficient force to damage the column from its healthy state. However, in a serve damage

state the shaker was observed to induce further damage. A total of 7 damage states were considered and these were labelled D0-D6 in the order of increasing damage. The healthy or no damage condition corresponded to D0, while D6 corresponded to severe damage with the removal of large chunks of concrete and exposure of reinforcing steel. In addition to the seven damage states three other states were considered. These states differed in mass: the additional masses were placed at the top of the column and designed to simulate a change in operating conditions. Two masses, either 2kg or 5kg, were added at D2 to give two new damage states D2 + 2kg and D2 + 5kg, respectively. Similarly, at D4 a mass of 5kg was added to give D4 + 5kg. Using a SDOF mass-spring formulation, the effective mass of the column together with the shaker was estimated to be 97kg and an additional 2kg or 5kg would equate to an increase in seismic mass of 2.1% or 5.2%, respectively. Table 4.14 gives a short description of the extent of damage in each damage state. Figure 4.10 shows photographs of the damage for states D1-D5. The numbers in the photographs denote faces and are used to refer to them in the subsequent discussions. No observable changes were seen between D5 and D6.

Table 4.14. Description of damage states for RC column.

Damage state	Description of damage
D0	No damage - healthy condition
D1	Minor damage – chipping 3-4mm depth across face 1
D2	Minor damage – additional chipping 3-4mm depth across face 3
D3	Moderate damage – additional chipping 5-10mm depth across face 1
D4	Severe damage – additional chipping 5-10mm depth across face 3, large pieces removed from corners exposing rebars
D5	Severe damage – additional pieces of concrete removed
D6	Severe damage – additionally damaged using shaker at full power

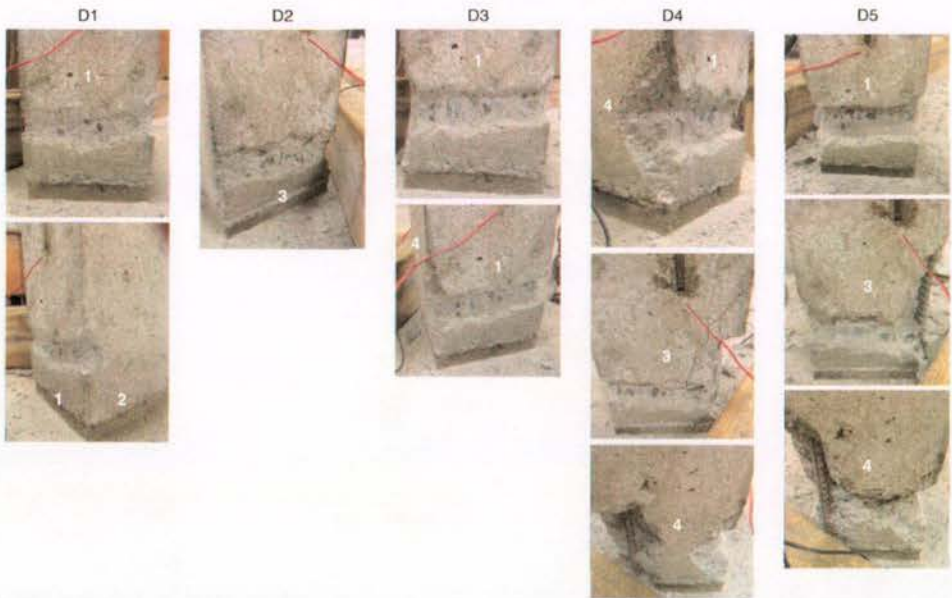


Figure 4.10. Photographs of RC column in damaged states D1-D5 numbered in the picture.

Modal analysis was conducted on the column in both healthy and damaged states using the ERA algorithm; refer to Chapter 3 for details. Of primary interest was the translational mode in the direction of forcing, later used to estimate the lateral stiffness of the column in each damage state. The ERA results were verified by analysing the peaks of the FRF curves. FRF curves were calculated using Welch's method with a Hanning window and no overlapping. Figure 4.11 shows the FRF curves obtained while the column was in damage states D0, D1, D4 and D6. Table 4.15 gives the natural frequencies and damping ratios for the translation mode in all damage states. The results show an immediate reduction in natural frequency for D1 and smaller changes amongst D1-D3. There was a larger drop in frequency between D3 and D4 indicating the severe extent of the damage while the separation between D4-D6 is again much closer. Generally, the FRF results are slightly greater than those estimated using ERA. Damping ratios appeared to increase initially with minor damage before decreasing with increasing damage severity, however, for D6 an increase was again observed. The frequencies for the three states with added mass, D2+2kg, D2+5kg, D6+5kg, are listed in Table 4.15 and show appreciable changes, placing these operational states into frequencies associated with more extensive damage. This would make damage detection based on natural frequencies alone inadequate.

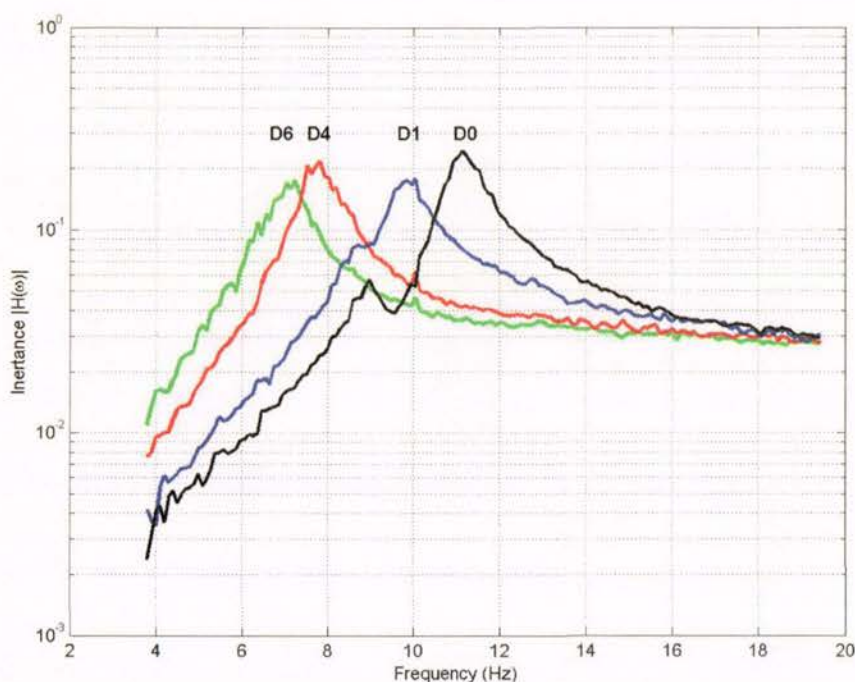


Figure 4.11. FRF curves for RC column in damage states D0, D1, D4 and D6.

Table 4.15. Frequency and damping ratio of translational mode of RC column at different damage states.

Damage state	Frequency (Hz)		ξ (%)
	ERA	FRF	ERA
D0	11.2	11.1	4.1
D1	9.88	9.77	6.8
D2	9.78	9.77	6.9
D3	9.67	9.57	5.8
D4	7.79	7.52	5.4
D5	7.58	7.52	4.6
D6	6.95	7.03	7.2
D2 + 2kg	9.54	-	-
D2 + 5kg	9.33	-	-
D4 + 5kg	7.48	-	-

At each damage state, a set of AR coefficients was obtained from two 30s white noise records produced under shaker excitation. A AR(30) model was chosen to fit the time histories and the coefficients were calculated using least squares from a 500-point window advancing 100 points until the end of the record was reached. Two separate cases were analysed: Case I and Case II. Case I contained the states D0-D6 and the 770-point data set was randomly divided into 600 and 170 points for training and testing, respectively. Case II contained the same data as Case I but also included the three other operational states with different masses. These states were lumped into their respective categories, e.g., D4+5kg lumped into D4 etc.. This increased the data set to 1100 points, which was divided into 900 and 200 points for training and testing respectively. The purpose of Case II study was to see if under changed operational conditions accurate damage classification was still possible.

The statistical distributions (histograms) of the 1st AR coefficient for damage states D0, D1, D4 and D6 are given in Figure 4.12. The figure shows that the mean of the 1st AR coefficient increases with damage.

4.7.1. Damage classification

Initially, the ANNs were trained to classify the damage into the states D0-D6. The states D0-D6 were assigned the vector outputs $[1\ 0\ 0\ 0\ 0\ 0\ 0]^T$ to $[0\ 0\ 0\ 0\ 0\ 0\ 1]^T$, respectively. A single hidden layer ANN with 10 hidden layer neurons was found to give adequate predictions. The results are shown in Table 4.16 with the number of actual and correctly classified cases for Case I. A total of 13 misclassifications (7.6%) were present which can be considered as a fairly good result. In Case II the three other operational states with different masses were also included as a challenge to the classification task and the results are given in Table 4.17. A total of 22 misclassifications (11.0%) were present. This increase in error was attributed to

the inclusion of the additional states, e.g., the D2 state had 10 errors alone. These results indicate that although reliable damage classification was still possible the accuracy decreased when additional masses were added.

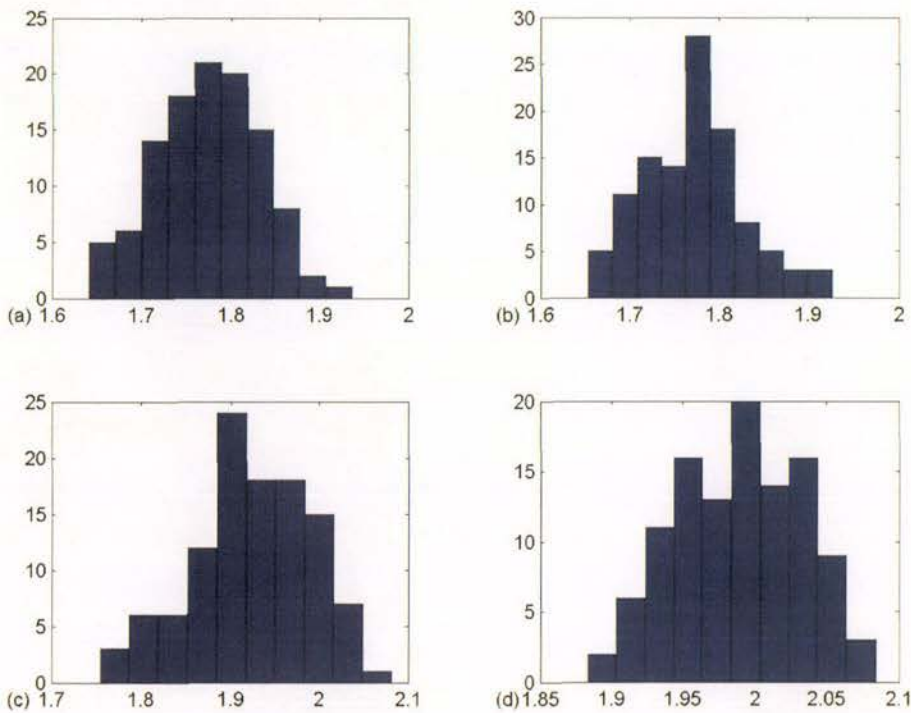


Figure 4.12. Histograms of the 1st AR coefficient from RC column in various damage states: (a) D0, (b) D1, (c) D4, and (d) D6.

Table 4.16. Case I classification results for RC column.

	D0	D1	D2	D3	D4	D5	D6
Actual number of data points	25	25	21	29	24	22	24
Correctly classified	25	18	18	29	22	22	23

Table 4.17. Case II classification results for RC column.

	D0	D1	D2	D3	D4	D5	D6
Actual number of data points	16	20	60	22	39	25	18
Correctly classified	14	14	50	21	37	25	17

4.7.2. Damage quantification

In this section the AR coefficients were related to the extent of damage, defined as the current remaining stiffness divided by the initial, undamaged stiffness. Table 4.18 gives the stiffness, estimated from a lumped mass-spring SDOF model and the damage for each state via model updating using natural frequencies, and percentage of remaining stiffness for each damage

state. In the severe damage cases the reduction in stiffness was approximately 50-60%, while in the minor cases the reduction was approximately 25%.

Table 4.18. Stiffness and damage at each state for RC column.

Damage state	Stiffness (N/m)	Percentage of remaining stiffness ^a
D0	4.75×10^5	100
D1	3.70×10^5	78
D2	3.63×10^5	76
D3	3.54×10^5	75
D4	2.30×10^5	48
D5	2.18×10^5	46
D6	1.83×10^5	39

^aBased on D0.

A single hidden layer ANN with 3 hidden layer neurons was chosen. The results are shown graphically in Figure 4.13 for Case I where the actual damage has been plotted against the detected damage. For perfect predictions the results would form seven points (0.39,0.39), (0.46,0.46), (0.48,0.48), (0.75,0.75), (0.76,0.76), (0.78,0.78) and (1.00,1.00). The figure shows good predictions over the broad range of damage. As before, the three remaining states with different masses can be included to simulate a change in operating conditions. The results for Case II are shown graphically in Figure 4.14. The figure shows more spread about the actual value of damage compared to Figure 4.13.

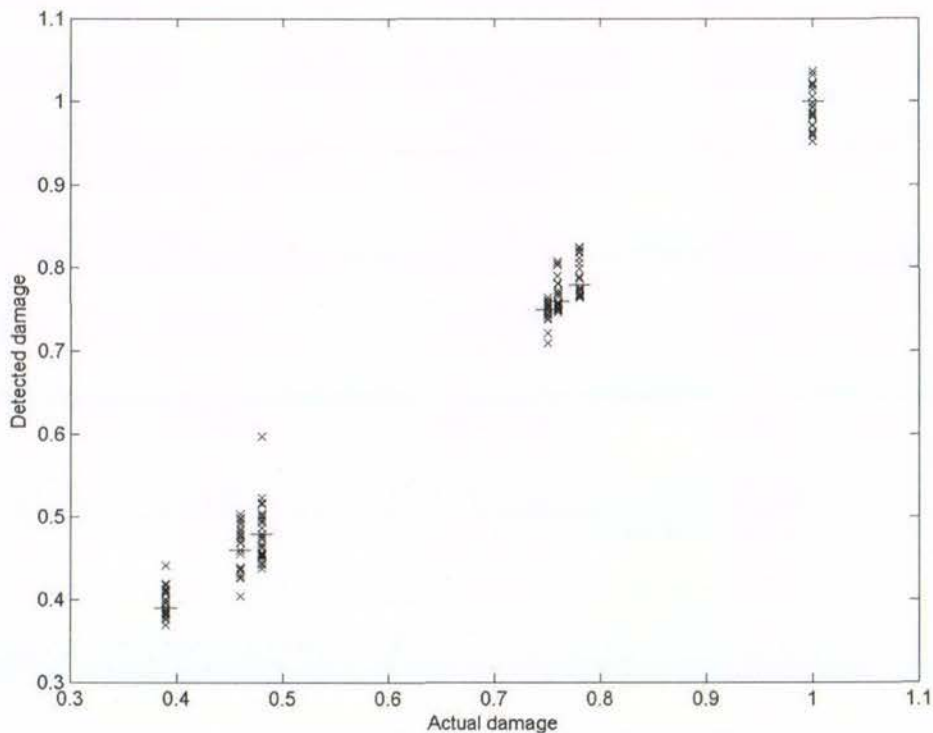


Figure 4.13. Detected vs. actual damage for RC column Case I using AR models.

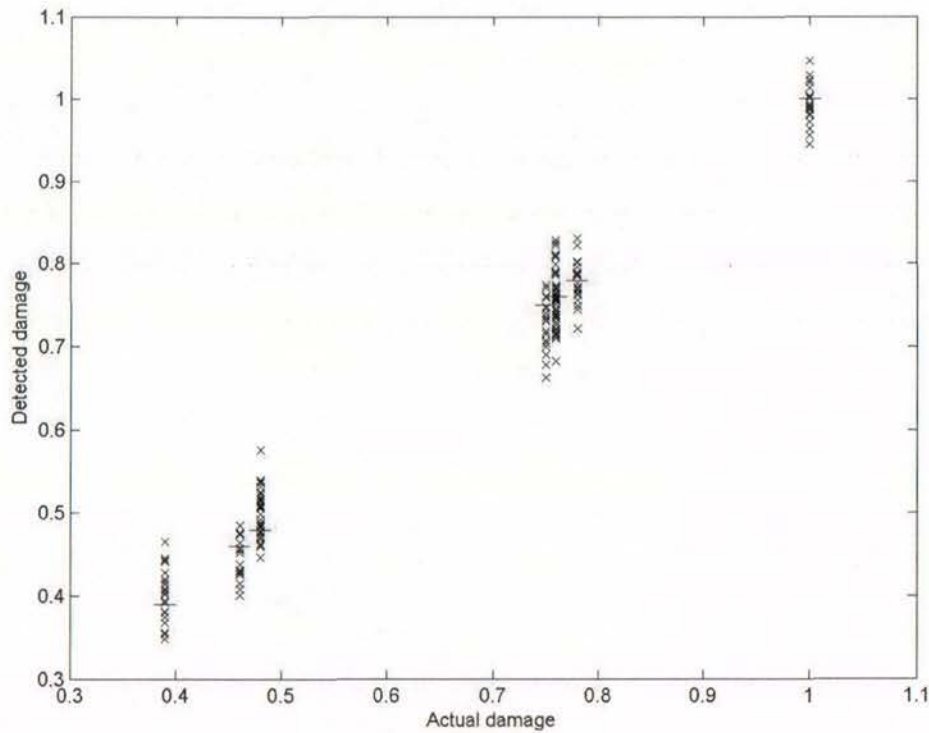


Figure 4.14. Detected vs. actual damage for RC column Case II using AR models.

Table 4.19 gives the means and two standard deviation bounds of the remaining stiffness identified by ANN for the two cases. The means were in good agreement, especially for Case I with the actual damage and at the 95% confidence level the errors are in most situations between 5-10%. For Case II the two standard deviations bounds were in most instances smaller than for Case I, however, the means were further away from the actual values.

Table 4.19. ANN identified damage as a percentage of remaining stiffness for RC column.

Damage state	Case I	Case II
D0	0.99 ± 0.05	0.99 ± 0.04
D1	0.79 ± 0.04	0.77 ± 0.03
D2	0.77 ± 0.04	0.76 ± 0.03
D3	0.75 ± 0.03	0.74 ± 0.03
D4	0.48 ± 0.07	0.48 ± 0.06
D5	0.46 ± 0.06	0.44 ± 0.04
D6	0.40 ± 0.03	0.42 ± 0.02

4.8. Conclusions

In this chapter, a damage detection and quantification method using time series models and ANNs was developed and applied to three experimental structures. Acceleration time histories from the structures in various damaged and undamaged states were fitted using AR and ARX models while the structures were excited dynamically. ANNs were used to interpret

changes in the AR coefficients and either classify damage into states or quantify the extent of damage as the percentage of remaining structural stiffness. Studies on the 3-storey bookshelf structure using output only AR models demonstrated that for stiffness reductions of 7-10%, the ANN was able to classify damage 100% correctly into states. Quantification and localisation of the damage, defined as the remaining stiffness, was achieved with small errors not exceeding 3.3% of the actual values at 95% confidence levels. This improved to 1.1% when input-output ARX models were used instead.

Due to computation limitations, ANNs were unable to be trained in some instances without reduction in dimensionality of the data. Two dimensionality reduction approaches were applied to the data from the ASCE Phase II Experimental SHM Benchmark Structure to reduce the dimension of the AR coefficient feature vector. Either a reduced number of AR coefficients and accelerometers were selected or the data was reduced using PCA. The results showed that significant reductions in the number of accelerometers and AR coefficients used can be achieved whilst maintaining good performance for damage classification. With judicious choice of accelerometers only about 5% of AR coefficients were required to obtain good or excellent damage classification results. For PCA-reduced data, using only 10 principal components (3.3% compared to the total number of AR coefficients) good classification (4.8% misclassifications) was achieved and excellent or perfect results were obtained using 30 principal components (10% compared to the total number of AR coefficients) with 100% correct classification. Overall data reduction via PCA provided a performance gain over selection of subset of AR coefficients and accelerometers.

Results from the RC column showed that the proposed method was able to classify and quantify damage despite changes in operating conditions, simulated by the addition of extra mass at the top of the column, although some deterioration of performance was observed. For classification tasks the number of misclassification increased from 7.6%, in the normal case to 11.0% for the changing operational conditions cases. Similarly, for damage quantification better agreement between the actual and detected damage was obtained for the normal case. The detected damage for the changing operational conditions case was once outside the two standard deviation bounds of the detected damage.

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CHAPTER 5

NEAREST NEIGHBOUR CLASSIFICATION, LEARNING VECTOR QUANTISATION AND SELF-ORGANISING MAPS FOR DAMAGE CLASSIFICATION AND QUANTIFICATION USING TIME SERIES ANALYSIS

Back-Propagation ANNs can be seen as ‘black boxes’ that require input data and produce classification results without much insight into their internal working. In this chapter, more intuitive analytical techniques based on clustering analysis that could be used for damage classification are studied: Nearest Neighbour Classification (NNC), Learning Vector Quantization (LVQ) and Self-Organizing Maps (SOM). The basic idea behind these methods is to establish a database of baseline damage features corresponding to various damage states, and later, when a new feature becomes available, assign it to the damage state with the closest distance between the new feature and the reference feature clusters. The NNC technique uses a fixed, predefined reference feature set, whereas LVQ adapts its reference feature set to optimise the classification results. NNC and LVQ are supervised learning techniques while SOM is an unsupervised one. Supervised methods differ from unsupervised methods in that learning is based on prior knowledge of the true classification of the data. Unsupervised methods attempt to classify data based on similarity between the data alone.

Initial inspection of the data to check the presence of damage clusters for quick visualisation of damage was conducted using two-dimensional projections obtained from PCA and Sammon mapping. For damage classification, the dimensionality of the AR coefficient data was reduced using PCA and NNC and LVQ were applied to classify damage into states. Clustering of the data using SOM was investigated and the results were analysed as a

classification task. Damage quantification was attempted by analysing the relative distance of data points to reference points of known damage extent.

5.1. Nearest Neighbour Classification

Nearest Neighbour Classification is a simple supervised pattern recognition technique. Given a set of fixed reference or codebook vectors $\mathbf{m}_i$ ($i = 1, \dots, k$) which have known classes, the input vector $\mathbf{x}$ is assigned to the class which the nearest $\mathbf{m}_i$ belongs. Several distance measures can be used including Manhattan, Euclidean, Correlation and Mahalanobis (Kohonen 1997). In this research the Euclidean or L_2 norm and the Mahalanobis distance measures were investigated. The Euclidean distance $D_E(\mathbf{x}, \mathbf{y})$ between two vectors $\mathbf{x}$ and $\mathbf{y}$ can be calculated using

$$D_E(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})^T (\mathbf{x} - \mathbf{y})} \quad (5.1)$$

The Mahalanobis distance $D_M(\mathbf{x}, \mathbf{y})$ between two vectors of the same distribution with a covariance matrix Σ can be calculated from

$$D_M(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})^T \Sigma^{-1} (\mathbf{x} - \mathbf{y})} \quad (5.2)$$

The Mahalanobis distance accounts explicitly for the different scales and correlations amongst vector entries and can be more useful in cases where these are significant. To reduce computational requirements the squared Euclidean or Mahalanobis distance may be used instead.

5.2. Learning Vector Quantisation

LVQ is a supervised machine learning technique designed for classification or pattern recognition by defining class borders. It is similar to NNC in that it uses a set of codebook vectors and seeks for the minimum of distances of an unknown vector to these codebook vectors as the criterion for classification. However, unlike in NNC where codebook vectors are fixed, an iterative procedure is adopted in which the position of the codebook vectors is adjusted to minimize the number of misclassifications. LVQ is a type of ANN and is closely related to vector quantisation and SOM. A complete description of LVQ can be found in Kohonen (1997) and only a brief discussion is given herein. LVQ are competitive networks and can be trained using several learning algorithms. The Optimised-Learning-Rate LVQ1

algorithm was used in this study (Kohonen 1997). This algorithm has an individual learning rate for each neuron or codebook vector, resulting in faster training.

Given a set of codebook vectors $\mathbf{m}_i$ ($i = 1, \dots, k$) which have been linked to each class region, the input vector $\mathbf{x}$ is assigned to the class which the nearest $\mathbf{m}_i$ belongs, i.e. NNC is performed. Let c define the index of the nearest $\mathbf{m}_i$ calculated using a certain distance measure. Learning is an iterative procedure in which the position of the codebook vectors is adjusted to minimise the number of misclassifications. At iteration step t let $\mathbf{x}(t)$ and $\mathbf{m}_i(t)$ be the input vector and codebook vectors, respectively. The codebook vectors are adjusted according to the following learning rule:

$$\begin{aligned}\mathbf{m}_c(t+1) &= [1 - s(t)\alpha_c(t)]\mathbf{m}_c(t) + s(t)\alpha_c(t)\mathbf{x}(t) \\ \mathbf{m}_i(t+1) &= \mathbf{m}_i(t) \text{ for } i \neq c\end{aligned}\quad (5.3)$$

$$s(t) = \begin{cases} +1 & \text{class correct} \\ -1 & \text{class incorrect} \end{cases} \quad (5.4)$$

$$\alpha_c(t) = \frac{\alpha_c(t-1)}{1 + s(t)\alpha_c(t-1)} \quad (5.5)$$

where $s(t)$ equals +1 or -1 depending if $\mathbf{x}(t)$ has been correctly or incorrectly classified and $\alpha_c(t)$ is the variable learning rate for codebook vector $\mathbf{m}_c$. Care must be taken to ensure that $\alpha_c(t) < 1$ for convergence. In this research an initial value of 0.3 was assumed for all $\alpha_i(t)$.

5.3. Self-Organising Maps

SOM attempt to create projections of high dimensional data in which the organisational structure, i.e. relative distances between adjacent data points is retained. SOM are often used for visualisation and clustering tasks. SOM is a type of ANNs and is similar to LVQ in that the position of the codebook vectors is adjusted iteratively. However, SOM is an unsupervised learning algorithm in which adjustment is based on the similarity between input and codebook vectors only, and class membership for the input does not need to be known. A formal discussion on SOM and self-organisation principals can be found in Kohonen (1997).

The formulation of SOM is similar to LVQ. Given a set of reference vectors $\mathbf{m}_i$ ($i = 1, \dots, k$) which have been initialised over the input space, the input vector $\mathbf{x}$ is assigned to the class

which the nearest $\mathbf{m}_i$ belongs. Let c define the index of the nearest reference vector i.e. $\mathbf{m}_c$. During training the $\mathbf{m}_i$ are adjusted according to the iterative application of

$$\begin{aligned}\mathbf{m}_c(t+1) &= \mathbf{m}_c(t) + \alpha(t)[\mathbf{x}(t) - \mathbf{m}_c(t)] \\ \mathbf{m}_i(t+1) &= \mathbf{m}_i(t) \quad \text{for } i \neq c\end{aligned}\tag{5.6}$$

where $\alpha(t)$ must satisfy $\lim_{t \rightarrow \infty} \alpha(t) \rightarrow 0$ for convergence. In this investigation $\alpha(k) = 0.5 / (0.1k + 1)$ was adopted, where k was the number of epochs, i.e. number of complete runs through the training data.

5.4. Sammon mapping

Sammon mapping (Sammon 1969) is a nonlinear transformation used for mapping a high dimensional space to a lower dimensional space in which local geometric relations are approximated. Consider a set of vectors $\mathbf{x}_i$ ($n = 1, \dots, n$) in a p -dimensional space and a corresponding set of vectors $\mathbf{y}_i$ in a lower dimensional q -space. For visualisation purposes q is usually chosen to be 2 or 3. The distance between vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ in p -space is given by

$$D_{ij}^* = D(\mathbf{x}_i, \mathbf{x}_j)\tag{5.7}$$

and the distance between the corresponding vectors $\mathbf{y}_i$ and $\mathbf{y}_j$ in q -space is

$$D_{ij} = D(\mathbf{y}_i, \mathbf{y}_j)\tag{5.8}$$

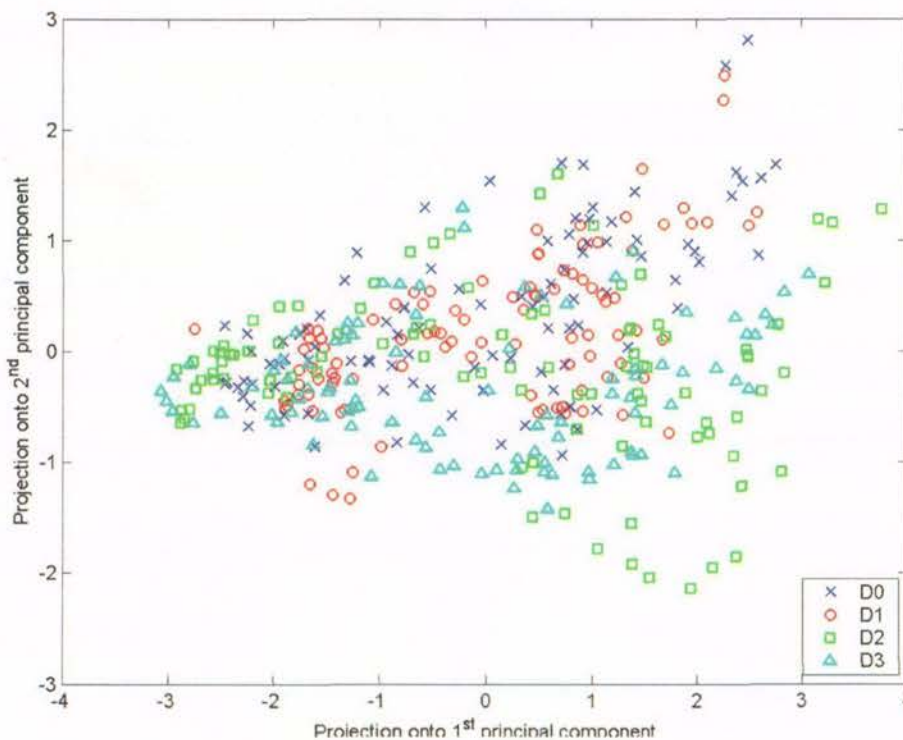
where D is a distance measure, usually the Euclidean distance. Mapping is achieved by adjusting the vectors $\mathbf{y}_i$ to minimise the following error function by steepest descent:

$$E = \frac{1}{\sum_{i=1}^n \sum_{j < i} D_{ij}^*} \sum_{i=1}^n \sum_{j < i} \frac{(D_{ij} - D_{ij}^*)^2}{D_{ij}^*}\tag{5.9}$$

5.5. Application to 3-storey bookshelf structure

In this section visualisation and statistical pattern recognition techniques were applied to the 3-storey bookshelf structure. The structure, experimental damage detection programme and data analysis have been described in Chapters 3 and 4. Using three univariate AR(12)

models, one for each storey acceleration resulted in a 36-dimensional feature vector. As a preliminary investigation to visualise and check the presence of clusters in the data, PCA, explained in Chapter 4, and Sammon mapping were used to create two-dimensional projections of the AR coefficient vectors. Projection of the data onto the first two principal components (Figure 5.1) showed no clearly defined clusters. In contrast, the Sammon map (Figure 5.2) showed some organisation of the data into overlapping bands, although again, no distinct clusters could be drawn. Using 3-dimensional mappings did not provide a better separation of clusters. These preliminary insights indicated that higher dimensional data was needed to separate the AR coefficients from the different damage states. For multidimensional data simple visual techniques were inadequate and more advanced approaches such as NNC and LVQ classification were needed.



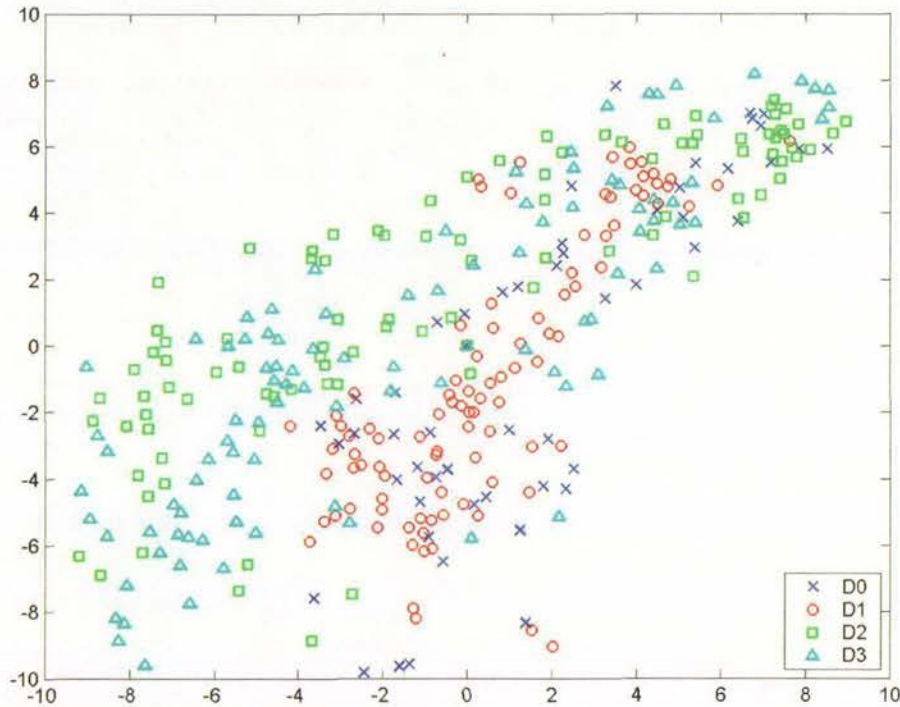


Figure 5.2. Projection of data from 3-storey bookshelf structure via Sammon mapping.

The two previously described pattern recognition techniques, NNC and LVQ, were used to classify damage into the states D0-D3. The feature dimension was reduced by projecting the AR coefficients onto the first 30, 20, 10 or 5 principal components using PCA. Only PCA was used for dimensionality reduction due to the computational effort required for Sammon mapping. The 388-point data set, consisting of 97 points from each damage state was randomly divided into 300 codebook vectors and 88 testing points, respectively. Five different random sets of codebook vectors were considered. Using NNC and averaging the results from the five runs, the obtained number of misclassifications and percentage errors using Euclidian and Mahalanobis distances are given in Table 5.1. The Mahalanobis distance measure outperformed the Euclidean by a considerable margin and adequate results with 6.8% misclassifications were obtained using 10 principal components. Increasing the number of principal components to 20 or 30 reduced the number of misclassification to 1.1%. The difference in performance between the Mahalanobis and Euclidean distance measures could be explained by the fact that the Mahalanobis distance accounts for the different scales of each principal component.

Table 5.1. Number and percentage of misclassifications using NNC for 3-storey bookshelf structure.

Number of principal components	Euclidean distance	Mahalanobis distance
30	17 (19.3%)	1 (1.1%)
20	16 (18.2%)	1 (1.1%)
10	16 (18.2%)	6 (6.8%)
5	27 (30.7%)	24 (27.3%)

Although NNC performed well, performance could be improved by using a more advanced and adaptive classification technique such as LVQ. The LVQ classification was used with the Mahalanobis distance measure and the same dimensionality reduction technique as above. The 388-point data set was divided into 300 points for training and 88 points for testing, respectively. The number of codebook vectors was chosen to be 30, 50 or 100. These were initialised by random selection from the training data set. The results averaged from five runs with different initialised codebook vectors are shown in Table 5.2. The table showed that increasing the number of codebook vectors and principal components reduced the number of misclassifications. For 20 or more principal components there was perfect classification. Overall a small improvement over NNC was observed.

Table 5.2. Number and percentage of misclassifications using LVQ classification for 3-storey bookshelf structure.

Number of principal components	Number of codebook vectors		
	30	50	100
30	0 (0%)	0 (0%)	0 (0%)
20	0 (0%)	0 (0%)	0 (0%)
10	14 (15.9%)	10 (11.4%)	5 (5.7%)
5	31 (35.2%)	31 (35.2%)	28 (31.2%)

Clustering using SOM was investigated on data reduced by both PCA and Sammon mapping using several different dimensions, however, SOM was unable to successfully identify the true classification of the data and typical misclassification rates of 50% were obtained. This poor performance was attributed to the degree of overlap as observed in the two-dimensional projections.

5.6. Application to ASCE SHM Phase II Experimental Benchmark Structure

The ASCE SHM Phase II Experimental Benchmark Structure is described in Chapter 3 and its simulated damage cases in Chapter 4. In this section, only damage configurations 1-7 (Table 4.12) were used for data visualisation and classification. The 805-point data set, consisting of AR coefficients from AR(20) models for all 15 sensors, contained 115 points from each damage configuration. Preliminary investigations showed that projection of the

data onto the first two principal components allowed distinct clustering in the data to be observed, see Figure 5.3. Three larger clusters were apparent that consisted of the configurations 2-4, configurations 1, 5-6 and configuration 7. Some overlapping existed between certain damage configurations, in particular configurations 1 and 5. This large-scale clustering could be due to similarities in the damage configurations. Referring to Table 4.12, configurations 2-4 may be considered to be similar in the extent of damage. Configuration 7 representing the un-braced case is significantly different from configurations 1-6. Using Sammon mapping, see Figure 5.4, a similar result was obtained, however, configuration 7 appeared to be further isolated from the other configurations. Both of these projections could be used for visual classification of the data once the damage clusters were clearly defined.

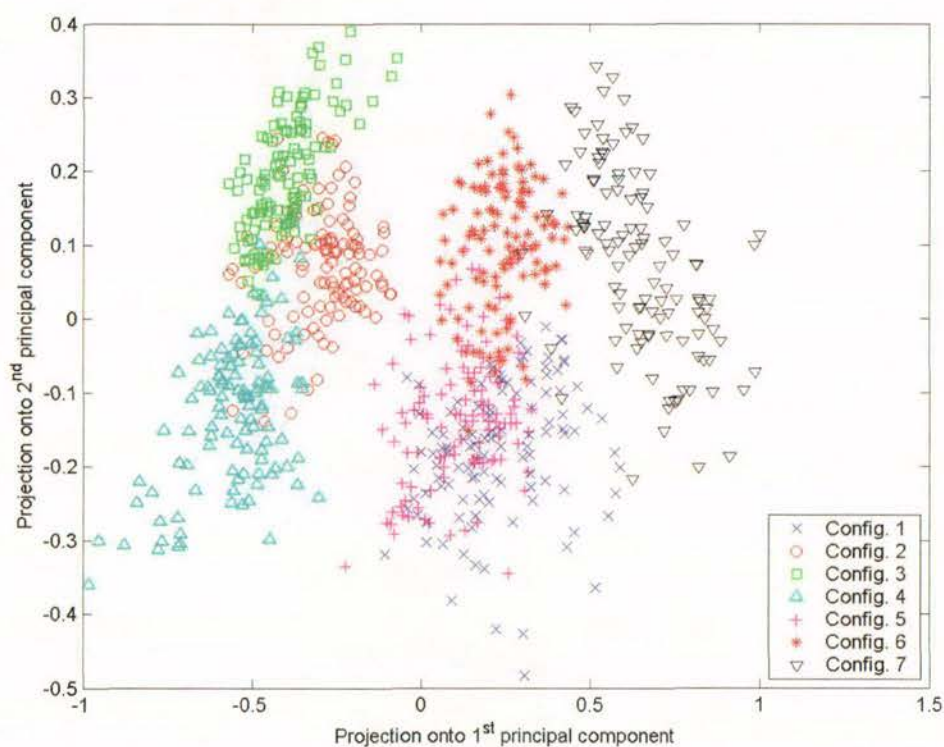


Figure 5.3. Projection of data from ASCE Phase II Experimental SHM Benchmark Structure on the first two principal components.

Table 5.3. Number and percentage of misclassifications using NNC for ASCE Phase II Experimental SHM Benchmark Structure.

Number of principal components	Euclidean distance	Mahalanobis distance
20	0 (0%)	0 (0%)
10	2 (1.0%)	2 (1.0%)
5	9 (4.5%)	9 (4.5%)
3	29 (14.5%)	27 (13.5%)

Using AR(20) models and all 15 accelerometers gave a set of 300 AR coefficients. This feature dimension was reduced by projection of the data onto the first 20, 10, 5 or 3 principal components. The 805-point data set was randomly divided into 605 codebook vectors and 200 testing points, respectively. Using NNC and averaging the results from five runs the number of misclassifications and percentage errors is given in Table 5.3. In this case, similar performance was obtained using both distance measures. Excellent performance was obtained using only 10 principal components and perfect classification was achieved using 20 principal components. These results show a significant reduction in dimensionality was achievable whilst maintaining good accuracy.

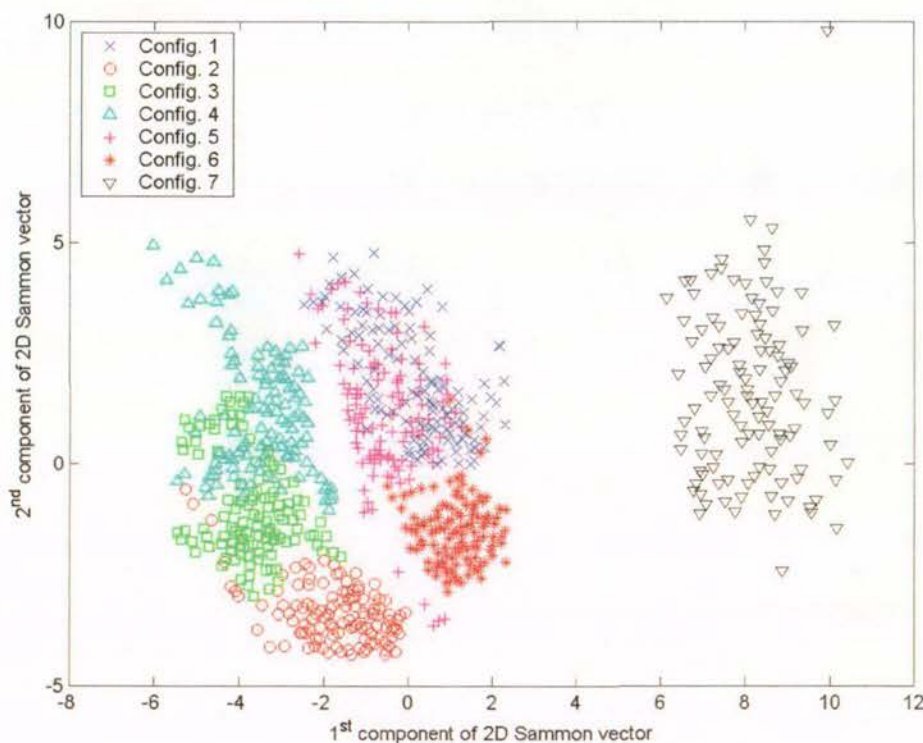


Figure 5.4. Projection of data from ASCE Phase II Experimental SHM Benchmark Structure via Sammon mapping.

LVQ classification was applied to PCA reduced data with the same number of components taken from above. The same sized training and testing data sets were used. The results from NNC showed that performance was similar for both distance measures, hence only the Euclidean distance was chosen for LVQ. The number of codebook vectors was chosen to be either 50, 100 or 200. These were initialised by random selection from the training set. The results obtained from averaging the number of misclassifications from five runs are shown in Table 5.4. Excellent performance with less than 1% misclassifications was obtained using 20 principal components for all numbers of codebook vectors. Good classification was still

achieved using 10 or 5 principal components, however, errors became significant once fewer than 5 components were used. Overall, performance was similar to NNC.

Table 5.4. Number and percentage of misclassifications using LVQ classification for ASCE Phase II Experimental SHM Benchmark Structure.

Number of principal components	Number of codebook vectors		
	50	100	200
20	1 (0.5%)	1 (0.5%)	0 (0%)
10	6 (3.0%)	3 (1.5%)	2 (1.0%)
5	10 (5.0%)	12 (6.0%)	7 (3.5%)
3	26 (13.0%)	26 (13.0%)	27 (13.5%)

Unlike the data from 3-storey bookshelf structure, Figures 5.3 and 5.4 clearly showed the presence of distinct damage clusters corresponding to specific damage configurations. Although SOM does not require a priori knowledge of which point belongs to which cluster, provided the true classification is known, as is the case in this study, the number of points misclassified to each cluster is an appropriate measure of performance of SOM. Using 7 reference vectors initialised randomly over the input space, a SOM was trained on the PCA reduced 805-point data set with 30 principal components. The number of reference vectors was adopted due to the known presence of 7 damage configurations in the data. Due to the unsupervised nature, no points were set aside for testing the SOM. Figure 5.5 shows the assigned cluster for each data point obtained from the SOM using the Euclidean distance. Compared to Figure 5.3 there is a clear resemblance, i.e. the SOM has correctly clustered the PCA reduced data into the actual damage configurations (note different symbols used for clusters in Figures 5.3 and 5.5). Table 5.5 shows a comparison between the true classification and the clustering obtained from SOM. In the table, the cluster number equates to the damage configuration that is most represented in each cluster. The table may best be analysed row by row. For perfect classifications nonzero entries would only appear on the diagonal. The table shows that damage configuration 7 was classified 100% correctly while adequate classification, 90% correct, was obtained for configurations 2 and 6. Results for configurations 3, 4 and 5 were rather mediocre with 67-77% corrects classifications. Configuration 1 was only 39% correctly classified and was the worst result obtained. Overall, there were 162 misclassifications or 20%. Compared to supervised classification the results were worse as a consequence of providing the classification algorithm with less information but were still quite promising.

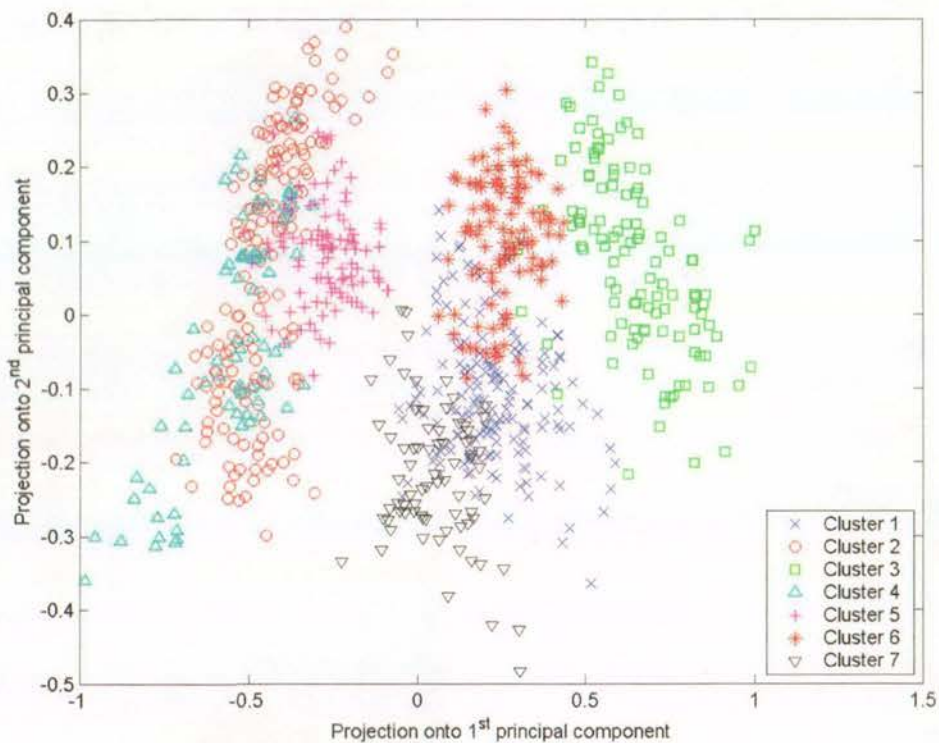


Figure 5.5. Clustering of data from ASCE Phase II Experimental SHM Benchmark Structure using SOM on PCA reduced data.

Table 5.5. Comparison of true classification and clustering from SOM on PCA reduced data for ASCE Phase II Experimental SHM Benchmark Structure.

Configuration	Cluster						
	1	2	3	4	5	6	7
1	45	0	0	0	70	0	0
2	0	104	0	11	0	0	0
3	0	0	88	27	0	0	0
4	0	0	77	38	0	0	0
5	30	0	0	0	85	0	0
6	1	0	0	0	10	104	0
7	0	0	0	0	0	0	115

Results of unsupervised classification of PCA reduced data were good, however, Figure 5.4 shows that better projections were obtained using Sammon mapping and therefore clustering of Sammon mapping reduced data was investigated. A 10 dimensional Sammon map was constructed and cluster analysis was performed using SOM with 7 reference vectors randomly initialised over the input space. Figure 5.6 shows graphically the first two components of the 10 dimensional Sammon vector and the assigned clusters. The classification results have been shown in Table 5.6 in which the assigned cluster and true damage configuration has been given. Once again the assigned cluster numbers have been relabelled for convenience of corresponding with damage configurations. Clusters

corresponding to damage configurations 2-3 and 6-7 were 100% correctly classified. Configurations 4 and 5 gave 90% and 82% correct results, respectively, which are marked improvements compared to using the PCA reduced data. Configuration 1 was again poorly classified with only 36% correct. The results show an overall improvement in results with a total of 10% misclassifications.

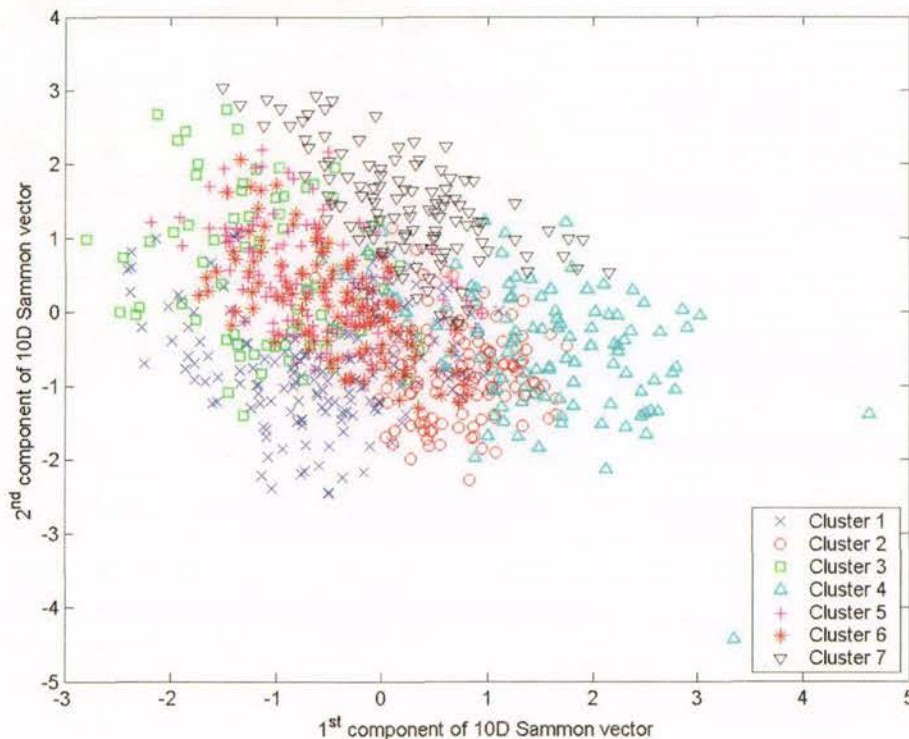


Figure 5.6. Clustering of data from ASCE Phase II Experimental SHM Benchmark Structure using SOM on Sammon projection data.

Table 5.6. Comparison of true classification and clustering from SOM on Sammon projection data for ASCE Phase II Experimental SHM Benchmark Structure.

Configuration	Cluster						
	1	2	3	4	5	6	7
1	41	0	0	0	65	9	0
2	0	115	0	0	0	0	0
3	0	0	115	0	0	0	0
4	12	0	0	103	0	0	0
5	21	0	0	0	94	0	0
6	0	0	0	0	0	115	0
7	0	0	0	0	0	0	115

5.7. Application to RC column

The RC column was described in Chapter 6. Due to only a single time series model being used, a AR(30) model, the data may be visualised by plotting the 1st AR coefficient against

the 2nd AR coefficient, see Figure 5.7. Note the Case I data set, i.e. without the additional masses were used here and in all but one analysis in this section (for details refer to Chapter 4). The figure shows the AR coefficients formed overlapping bands with the undamaged state D0 and the most extensive damage state D6 being the furthest apart. This property would be later investigated in an attempt to quantify damage.

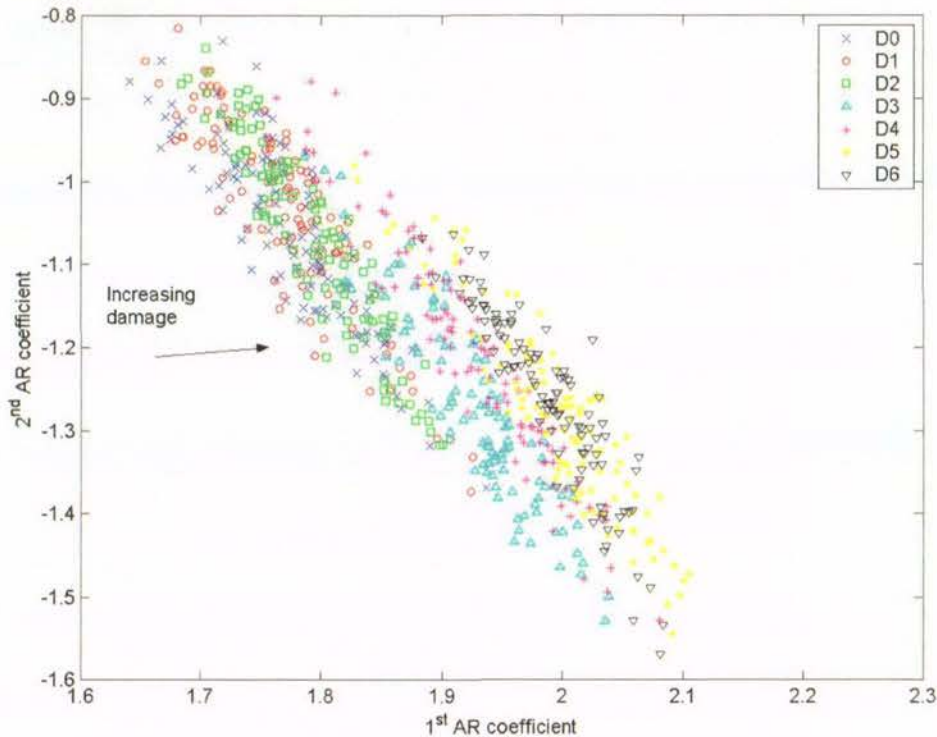


Figure 5.7. Scatter plot of the 1st vs. 2nd AR coefficients for RC column in all Case I damage states.

5.7.1. Damage classification

Column damage was classified into the states D0-D6 using NNC. Two approaches for data reduction were compared: either selection of a subset of AR coefficients or reduction using PCA. The 770-point data set was randomly divided into 600 points for codebook vectors and 170 points for testing, respectively. The results were averaged from five runs. The number of misclassifications and percentage errors using either the Euclidean or Mahalanobis distance measures and both data reduction techniques are given in Table 5.7. The Mahalanobis distance measure consistently performed better than the Euclidean measure. Both dimensionality reduction approaches, PCA and Sammon mapping, gave similar good or excellent results with only around 1% of misclassifications when more than 20 features were used. When using 10 features, i.e. either the first 10 AR coefficients or principal components,

differences in performance were observed and the subset AR coefficient data performed better with 9.4% misclassifications compared to 17.1% for the PCA reduced data. Overall good classification results were obtained.

Table 5.7. Number and percentage of misclassifications using NNC for Case I data from RC column.

Number of coefficients/components	Subset of AR coefficients		Principal components	
	Euclidean distance	Mahalanobis distance	Euclidean distance	Mahalanobis distance
30	15 (8.8%)	2 (1.2%)	16 (9.4%)	1 (0.6%)
20	29 (17.1%)	2 (1.2%)	23 (13.5%)	2 (1.2%)
10	74 (43.5%)	16 (9.4%)	50 (29.4%)	29 (17.1%)

LVQ classification was applied to the AR coefficient data with the Mahalanobis distance only. The 770-point data set was randomly divided into 600 points for training and 170 points for testing, respectively. Either 50, 100 or 200 codebook vectors were used. These were initialised by random selection from the training set. The results obtained from averaging the number of misclassifications from five runs are shown Table 5.8. The performance of LVQ was similar to NNC and the best results were obtained using 30 AR coefficients and 100 or 200 codebook vectors.

Table 5.8. Number and percentage of misclassifications using LVQ for Case I data from RC column.

Number of coefficients	Number of codebook vectors		
	50	100	200
30	6 (3.5%)	2 (1.2%)	2 (1.2%)
20	9 (5.3%)	5 (2.9%)	4 (2.4%)
10	31 (18.2%)	27 (15.9%)	20 (11.8%)

Using the Case II data set that included damage states with additional mass added to the top column (see Chapter 4) NNC and LVQ classification was applied using the Mahalanobis distance measure. The 1100-point data set was divided into 900 codebook vectors and 200 testing points for NNC. For LVQ the same number of training and testing points were used and the number of codebook vectors was chosen to be 100. The results for both NNC and LVQ are shown in Table 5.9. Performance was once again similar, however, LVQ did give the best classifications. Overall performance was similar to the results obtained using the Case I data with approximately 1% misclassification for the best result.

Table 5.9. Number and percentage of misclassifications using NNC and LVQ classification on RC column Case II.

Number of coefficients	NNC	LVQ
30	3 (1.5%)	2 (1.0%)

20	3 (1.5%)	2 (1.0%)
10	14 (7.0%)	9 (4.5%)

Unsupervised classification using SOM was investigated, however the results were poor and have not been shown here.

5.7.2. Damage quantification

In the foregoing discussions only damage classification has been studied, however, damage quantification may also be attempted. It was proposed that the centroids of the AR coefficient clusters corresponding to each damage state would correspond to the state's damage severity (relative remaining stiffness), e.g. the centroid of cluster D0 would correspond to a damage of 1.00, the centroid of cluster D1 to a damage of 0.78, etc. The assumption was made that the distance between a data point and the 7 damage centres was related to the likelihood of the data point having that particular damage severity, e.g. if the data point was closest to the D1 centroid it was most likely to have a damage of 0.78, however, it was also possible, although with smaller probability, that the point corresponded to a different damage severity. To account for this, the distances between the data point and damage state centroids were combined in a membership function that gave more weight to the closest damage centre. Membership m_i or closeness of each data point $\mathbf{x}$ to each of the 7 damage centroids $\mathbf{c}_i$ ($i = 0, \dots, 6$) was calculated using a Gaussian function:

$$m_i = \exp\left(\frac{-(\mathbf{x} - \mathbf{c}_i)^T (\mathbf{x} - \mathbf{c}_i)}{c}\right) \quad (5.10)$$

where c was a constant determined by trial and error to yield the best damage quantification results. Damage was estimated as the normalised sum:

$$damage = \frac{1.00m_0 + 0.78m_1 + 0.76m_2 + 0.75m_3 + 0.48m_4 + 0.46m_5 + 0.39m_6}{m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6} \quad (5.11)$$

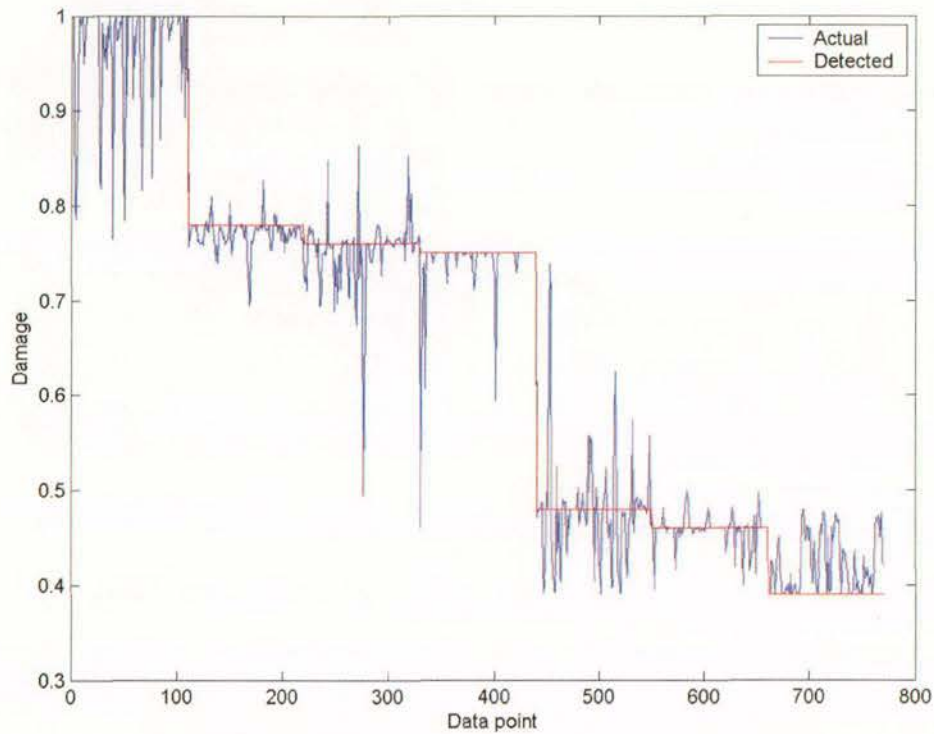


Figure 5.8. Damage quantification results for RC column.

Table 5.10. Detected mean damage for RC column.

	D0	D1	D2	D3	D4	D5	D6
Actual	1.00	0.78	0.76	0.75	0.48	0.46	0.39
Mean detected	0.97	0.77	0.75	0.74	0.48	0.46	0.42

Using the Case I data set, the centroids of the 7 damage states were obtained and the Mahalanobis distance between these centroids and the data points was calculated. The data set was not divided into training and testing data sets as this approach was not a learning technique. A trial and error approach was taken to determine the best number of AR coefficients and the constant c . The best results were obtained using the full set of 30 AR coefficients with a value of $c = 25$. These results have been shown graphically in Figure 5.8 in which the actual damage has been shown against the detected damage. Table 5.10 gives the actual damage and the mean damage detected for each state. Due to the constraint of the states D0 and D6 having damage severities of 1.00 and 0.39 respectively, the maximum and minimum damage obtainable using Equation (5.11) was also 1.00 and 0.39. This caused the mean damage detected for these states to be lower for D0 and higher for D6, respectively than their true values. However, for the remaining damage states the mean damage detected showed good agreement with the actual value. At the same time, the spread of results was in

some cases considerable, e.g. state D4, and a number of outliers was clearly seen, e.g. states D0, D2 and D4.

5.8. Conclusions

Three techniques for damage classification based on clustering analysis, namely NNC, LVQ and SOM were studied in this chapter. The basic idea behind these methods is to establish a database of baseline damage features corresponding to various damage states, and later, when a new feature becomes available, assign it to the damage state with the closest distance between the new feature and the reference feature clusters. NNC and LVQ are supervised learning techniques while SOM is an unsupervised one.

In techniques based on cluster analysis it is convenient to visualise high dimensional data for quick inspection of cluster presence. Two-dimensional projections were obtained using PCA and Sammon mapping. Organisation of the data into clusters corresponding to damage states with some separation between them was observed especially with the ASCE Phase II Experimental SHM Benchmark Structure data in which distinct clusters belonging to specific damage configurations could clearly be seen. Another benefit from using projection technique is a reduction in data size and smaller computational demand.

NNC and LVQ were applied to damage classification in the three experimental structures. The dimensionality of the AR feature vector was reduced by projection onto a fewer number of principal components. Results showed that significant reductions were possible whilst maintaining good classification. Generally, LVQ gave a slight performance advantage over NNC. For the 3-storey bookshelf structure classification based on the Mahalanobis distance was superior to the Euclidean distance. Excellent or perfect results, 1% or 0% misclassifications, were obtained using 20 principal components with the Mahalanobis distance and using NNC and LVQ, respectively. Classification results for the ASCE Phase II Experimental SHM Benchmark Structure were similar for both distance measures and classification techniques. Excellent or perfect results, 1% or 0% misclassifications, was obtained using both NNC and LVQ with 20 principal components. Noting that original feature vector had a dimension of 300, its dimensionality was reduced by a factor of 15. Similar results between NNC and LVQ were also obtained for the RC column data, however, the Mahalanobis distance again performed better than the Euclidean distance. Using 30 AR

coefficients not transformed using PCA, NNC and LVQ both had only 1.2% misclassifications.

Overall, the performance of NNC and LVQ classification was comparable to results obtained using BP ANNs. Damage in the 3-storey bookshelf structure was 100% correctly classified by the BP ANN. NNC and LVQ classified respectively 99% and 100% of the data correctly. Better results were obtained using NNC and LVQ classification in the RC column. For Case I data LVQ had 1.2% misclassifications compared to 7.6% using BP ANNs and 1.0% compared to 11.0% for Case II data. As only 7 out of 9 configurations were analysed for the ASCE Phase II Experimental SHM Benchmark Structure a comparison with the BP ANN results could not be easily drawn, although all classification techniques worked well on this data.

The unsupervised clustering technique of SOM was applied successfully to the ASCE Phase II Experimental SHM Benchmark Structure data. Using either PCA or Sammon mapping reduced data 20% or 10% misclassifications were obtained, respectively. Several damage configurations were identified with 100% correct results. These findings indicate that SOM is a promising technique. For the other structures, however, the results were worst and produced unacceptable classification results because of significant cluster overlaps.

Damage quantification was attempted on the RC column structure using a weighted distance between data and the centroids of damage clusters corresponding to known stiffness degradations. The results showed the mean detected values of damage were in good agreement with the actual values, but the spread of result was considerable.

5.9. References

- Kohonen, T. (1997). Self-organizing maps. Berlin, Springer.
- Sammon, J. J. W. (1969). "Nonlinear mapping for data structure analysis." IEEE Transactions on Computers C-18(5): 401-409.

CHAPTER 6

ONLINE DAMAGE DETECTION USING RECURSIVE IDENTIFICATION OF TIME SERIES MODELS AND BACK-PROPAGATION ARTIFICIAL NEURAL NETWORKS

In previous chapters, damage was detected using offline procedures in which time series models were estimated from acceleration data recorded over a certain time interval. In this chapter, an extension to online or real-time damage detection was developed using recursive identification techniques to estimate the time series models online. Online damage detection would allow damage to be detected and tracked as it accumulates over the period of ground motion duration.

Two recursive techniques were chosen for AR model identification from structural accelerations: the forgetting factor and the Kalman filter. Initially, a simple analytical linear 3-DOF lumped mass-spring model representing a shear building was investigated with damage simulated as sudden stiffness loss. BP ANNs were trained to interpret the changes in the AR coefficients in order to trace online stiffness degradation. The effect of measurement noise on damage prediction was assessed with the addition of Gaussian white noise to the analytical time history data. Later, analytical models of a 1-DOF nonlinear elastoplastic oscillator and a more complex 3-DOF nonlinear Bouc-Wen hysteretic structure were investigated and the detection of nonlinear damage was addressed. Using online identification of AR models the presence or on-set of nonlinearity could be detected by observation of sudden changes in the AR coefficients. The effect of noise on nonlinearity on-set detection was also studied.

6.1. Recursive identification of AR time series

Recursive parameter estimation techniques for the identification of times series models include the use of the forgetting factor, Prediction Error Method (PEM) and Kalman filter approaches (Ljung 1999). These techniques are suitable for the identification of systems in which the model coefficients are expected to vary over time. In this chapter, the forgetting factor and Kalman filter approach were investigated. For AR time series the PEM reduces to the forgetting factor approach.

6.1.1. Forgetting factor

The forgetting factor approach (Ljung 1999) is a recursive algorithm that gives a weighted least-squares estimate. The forgetting or weighting factor λ determines the method's ability to track changes over time by assigning more weight to current observations. This parameter is usually taken to be $0.98 < \lambda < 0.995$. The algorithm estimates the model time varying parameters θ_t , in this case the AR coefficients, at time step t from recursive application of the following equations

$$\theta_t = \theta_{t-1} + \mathbf{L}_t [y_t - \phi_t^T \theta_{t-1}] \quad (6.1)$$

$$\mathbf{L}_t = \frac{\mathbf{P}_{t-1} \phi_t}{\lambda + \phi_t^T \mathbf{P}_{t-1} \phi_t} \quad (6.2)$$

$$\mathbf{P}_t = \frac{1}{\lambda} \left[\mathbf{P}_{t-1} - \frac{\mathbf{P}_{t-1} \phi_t \phi_t^T \mathbf{P}_{t-1}}{\lambda + \phi_t^T \mathbf{P}_{t-1} \phi_t} \right] \quad (6.3)$$

where y_t is the current observed value of the time series and $\phi_t = [y_{t-1}, \dots, y_{t-na}]^T$ is a vector of previous values. The gain $\mathbf{L}_t$ determines how much the prediction error affects the update of the parameter estimate. Matrix $\mathbf{P}_t$ is the estimated covariance matrix. In this study, θ_0 and $\mathbf{P}_0$ were initialised with $\theta_0 = \mathbf{0}$ and $\mathbf{P}_0 = 1 \times 10^4 \mathbf{I}$, where $\mathbf{0}$ and $\mathbf{I}$ are the null and identity matrices of appropriate sizes.

6.1.2. Kalman filter

The Kalman filter (Harvey 1989) is an optimal linear recursive estimator. Consider the following state space model

$$\mathbf{y}_t = \mathbf{Z}_t \boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t \quad (6.4)$$

$$\boldsymbol{\alpha}_t = \mathbf{T}_t \boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta}_t \quad (6.5)$$

Equation (6.4) is referred to as the measurement equation and Equation (6.5) as the transition equation, where $\mathbf{y}_t$ is the vector of outputs and $\boldsymbol{\alpha}_t$ is the state vector. Matrices $\mathbf{Z}_t$ and $\mathbf{T}_t$ are the measurement and transition matrices respectively. Vectors $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\eta}_t$ represent noises with zero mean multivariate Gaussian distributions and covariance matrices $\mathbf{H}_t$ and $\mathbf{Q}_t$, respectively. These noises can also be contemporaneously correlated so that

$$E(\boldsymbol{\varepsilon}_t \boldsymbol{\eta}_t^T) = \begin{cases} \mathbf{G}_t, & t = s \\ \mathbf{0}, & t \neq s \end{cases} \quad (6.6)$$

where E denotes the expected value operator.

The optimal estimation $\mathbf{a}_t$ of the state vector $\boldsymbol{\alpha}_t$ conditional on the information available at time t can be obtained through recursive application of the following prediction equations

$$\mathbf{a}_{t|t-1} = \mathbf{T}_t \mathbf{a}_{t-1} \quad (6.7)$$

$$\mathbf{P}_{t|t-1} = \mathbf{T}_t \mathbf{P}_{t-1} \mathbf{T}_t^T + \mathbf{Q}_t \quad (6.8)$$

and updating equations

$$\mathbf{a}_t = \mathbf{a}_{t|t-1} + (\mathbf{P}_{t|t-1} \mathbf{Z}_t^T + \mathbf{G}_t) \mathbf{F}_t^{-1} [\mathbf{y}_t - \mathbf{z}_t(\mathbf{a}_{t|t-1})] \quad (6.9)$$

$$\mathbf{P}_t = \mathbf{P}_{t|t-1} - (\mathbf{P}_{t|t-1} \mathbf{Z}_t^T + \mathbf{G}_t) \mathbf{F}_t^{-1} (\mathbf{Z}_t \mathbf{P}_{t|t-1} + \mathbf{G}_t^T) \quad (6.10)$$

with

$$\mathbf{F}_t = \mathbf{Z}_t \mathbf{P}_{t|t-1} \mathbf{Z}_t^T + \mathbf{Z}_t \mathbf{G}_t + \mathbf{G}_t^T \mathbf{Z}_t^T + \mathbf{H}_t \quad (6.11)$$

Matrix $\mathbf{P}_t$ is the covariance matrix of the estimation error

$$\mathbf{P}_t = E \left[(\boldsymbol{\alpha}_{t-1} - \mathbf{a}_{t|t-1}) (\boldsymbol{\alpha}_{t-1} - \mathbf{a}_{t|t-1})^T \right] \quad (6.12)$$

Equations (6.7)-(6.12) represent the Kalman filter.

Applying the Kalman filter to the identification of AR coefficients, the output and state vectors are

$$\mathbf{y}_t = \mathbf{y}_t \quad (6.13)$$

$$\boldsymbol{\alpha}_t = \boldsymbol{\theta}_t \quad (6.14)$$

Assuming the coefficients vary according to a random walk model the transition equation is

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta}_t \quad (6.15)$$

The rationale behind the choice of the random walk model was that in an undamaged system the AR coefficients do not change with time except for some stochastic uncertainty in their identification. Selection of $\mathbf{Q}_t$ allows the tracking of the Kalman filter to changes in the coefficients to be adjusted. In this study, $\boldsymbol{\theta}_0$ and $\mathbf{P}_0$ were initialised with $\boldsymbol{\theta}_0 = \mathbf{0}$ and $\mathbf{P}_0 = 1 \times 10^8 \mathbf{I}$, where $\mathbf{0}$ and $\mathbf{I}$ are the null and identity matrices of appropriate sizes.

6.2. Application to an analytical linear 3-DOF lumped mass-spring building model

A linear 3-DOF lumped mass-spring shear building model (Figure 6.1) was used in the numerical investigations. The lateral stiffness of each storey for the undamaged structure were set to $k_1 = k_2 = k_3 = 1 \times 10^7$ N/m and the lumped storey masses were set to $m_1 = m_2 = m_3 = 1 \times 10^4$ kg. A Rayleigh damping model was used and damping was set at 5.0% critical for the 1st and 2nd modes and the 3rd mode had 6.2% damping. The natural frequencies of the structure were 2.24Hz, 6.28Hz and 9.07Hz for the 1st, 2nd, and 3rd modes, respectively.

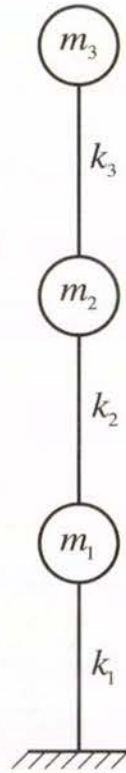


Figure 6.1. Linear 3-DOF lumped-mass model.

Damage in the model was simulated as steep ramped drops in storey stiffness occurring at predetermined times. Stiffness of the 1st storey was reduced from 1.00 to 0.80 of the initial, undamaged value and afterwards from 0.8 to 0.4. Stiffness of the 2nd storey was reduced from 1.00 to 0.70 and stiffness of the 3rd storey from 1.00 to 0.90. In all cases, the stiffness was linearly interpolated over a 2s time period. These stiffness drops are illustrated in Figures 6.2 and 6.3 as the continuous blue line representing the actual stiffness.

For the purpose of training a BP ANN seven damage severities (relative remaining stiffnesses) were considered: 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0. Considering various combinations of these severities occurring at the different stories resulted in $7^3=343$ damage states. Models with so reduced stiffness were excited by white noise ground motion. Three univariate AR(30) models were identified from the accelerations of the three stories using either the forgetting factor or Kalman filter approaches. Two AR coefficient training data sets were constructed, one for each approach, containing AR coefficients for different damage scenarios. A set of 100 AR coefficients was obtained for each of the 343 states. For both approaches, a hidden layer BP ANN with 3 hidden layer neurons was trained. The following sections show the results of each recursive identification method.

6.2.1. Forgetting factor approach

Using the forgetting factor approach, the ability of the method to track changes can be tuned using the forgetting factor λ . Smaller values of λ result in 'shorter memory', which makes quick identification of abrupt changes possible but at the expense of larger stochastic variation in the results. Larger values cause the algorithm to react with delay to sudden changes in the system being identified but result are smoother. After some initial trials a value of $\lambda = 0.99$ was adopted for all three models as a trade off. Figure 6.2 shows the damage detected in each storey compared to the simulated value. Note the large error in the initial start up phase was due to the AR coefficients initialised as zero. The figure shows that the forgetting factor approach was effective at tracking damage and scatter about the exact values was small.

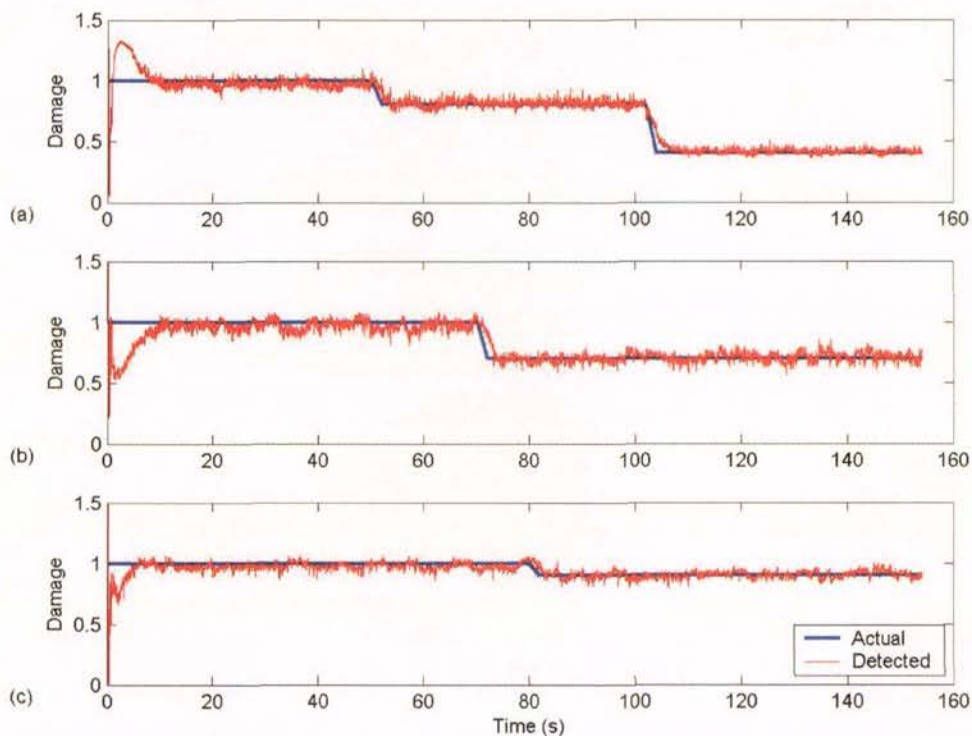


Figure 6.2. Damage detection using forgetting factor $\lambda = 0.99$: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

6.2.2. Kalman filter approach

The tracking abilities of the Kalman filter can be tuned by selection of the matrix $\mathbf{Q}$. After initial trials $\mathbf{Q} = 1.5\mathbf{I}$ for all three models was adopted. Figure 6.3 shows the detected damage

at each storey compared to the simulated damage. The figure shows generally a slower response to changes in the structure than in Figure 6.2. Also, in the 3rd storey the detected damage appeared to not track the simulated damage. One problem with the Kalman filter was the number of parameters required to define the filter for efficient tracking. In this case, three 30×30 matrices were required whereas for the forgetting factor approach, only three forgetting factors were required.

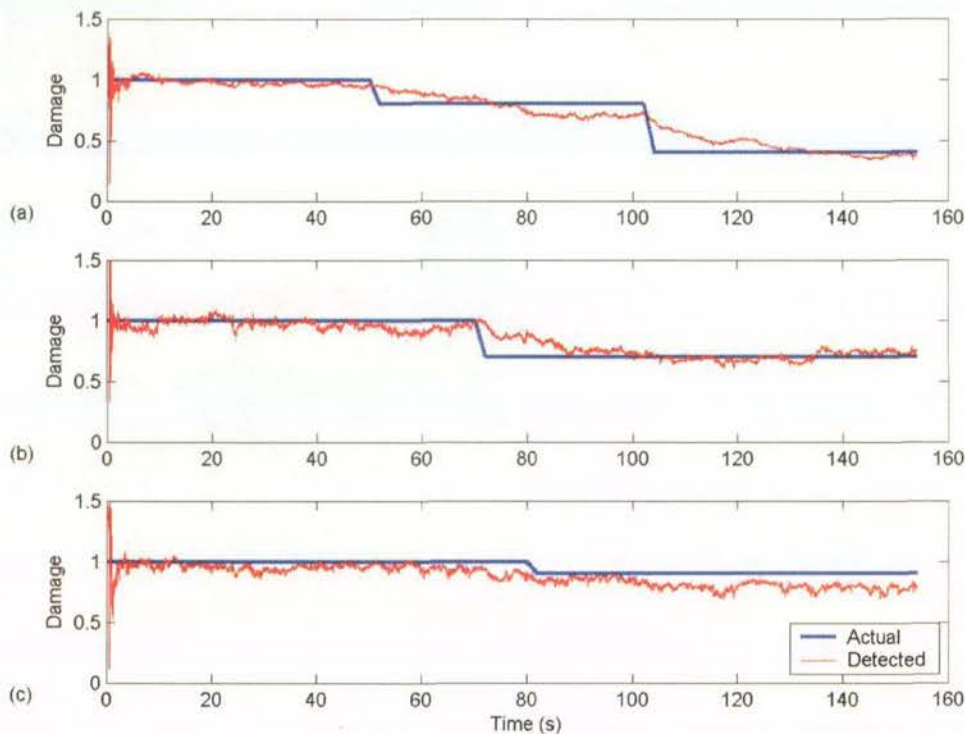


Figure 6.3. Damage detection using Kalman filter: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

6.2.3. Effect of measurement noise

To assess the effect of measurement noise on the online damage detection method, Gaussian white noise was added to the acceleration time histories. Two noise-to-signal ratios of 2% or 5%, respectively, were investigated. Previous results showed that the forgetting factor approach was more effective at tracking changes in the system than the Kalman filter and was therefore chosen here. Figure 6.4 shows the detected damage and simulated damaged once 2% noise was added to the acceleration time histories of each storey. There was a clear increase in the degree of scatter about the exact stiffness values compared to Figure 6.2. Despite this the actual values were still tracked and good damage quantifications were

obtained. Figure 6.5 shows the results with 5% noise. Once again there was an increase, although somewhat smaller, in scattering about the simulated values. However, the actual damage was still tracked and damage quantification produced adequate results.

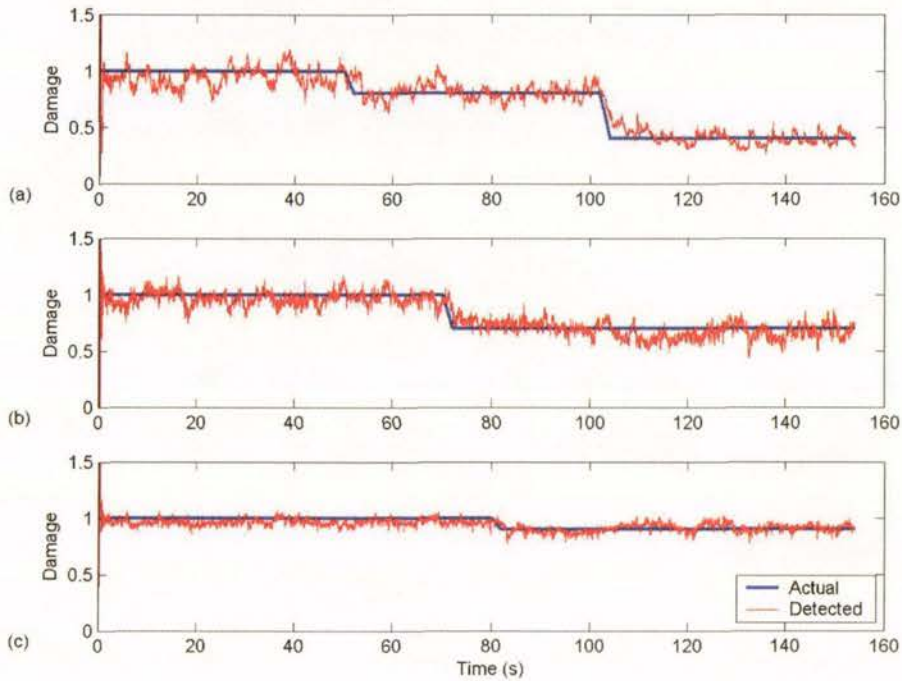


Figure 6.4. Damage detection with 2% noise using forgetting factor $\lambda = 0.99$: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

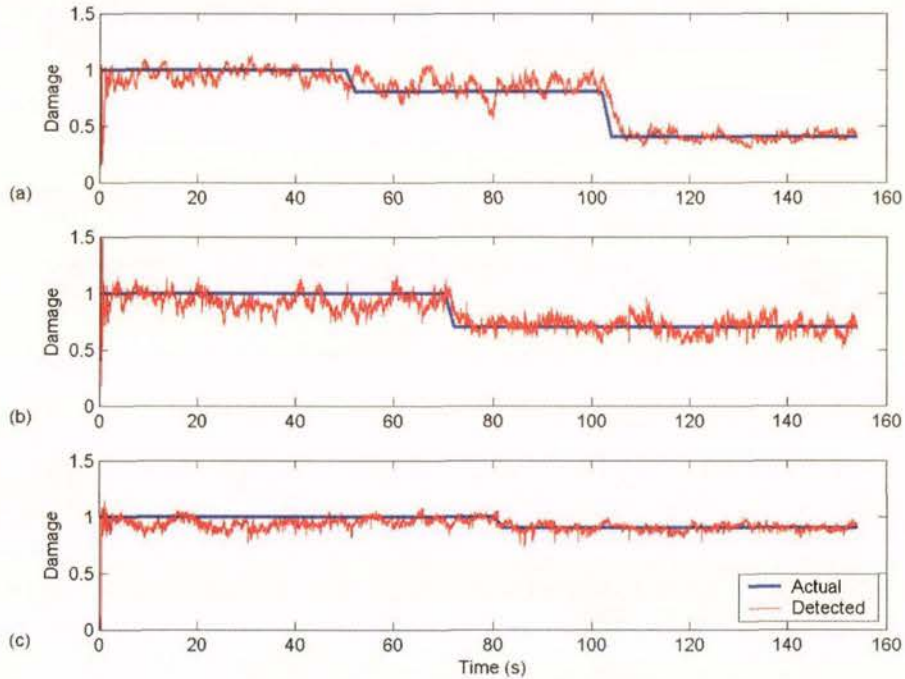


Figure 6.5. Damage detection with 5% noise using forgetting factor $\lambda = 0.99$: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

6.3. Application to an analytical model of a 1-DOF elastoplastic oscillator

Research up to this point has dealt with the detection of damage in linear structures in which damage was defined as a change in lateral stiffness. However, the response of a damaged structure during a strong earthquake will most likely be nonlinear and the concept of damage quantification as a reduction of lateral stiffness becomes inappropriate. To illustrate this, consider a simple elastoplastic system representing the idealised behaviour of a steel structure in which the tangent stiffness is initially k until the yield displacement is reached and the tangent stiffness drops to zero. Figure 6.6 shows a hysteresis loop of such a model with a yielding force of 2×10^4 N, an initial tangent stiffness of 5×10^5 N/m and a yield displacement of ± 0.04 m. During an earthquake the structure may yield, however, once the large vibrations have passed the structure returns to the initial tangent stiffness. Clearly, damage has occurred and yet quantification by a reduction in lateral stiffness is inadequate. A new indicator of damage taking into account the response nonlinearity is required.

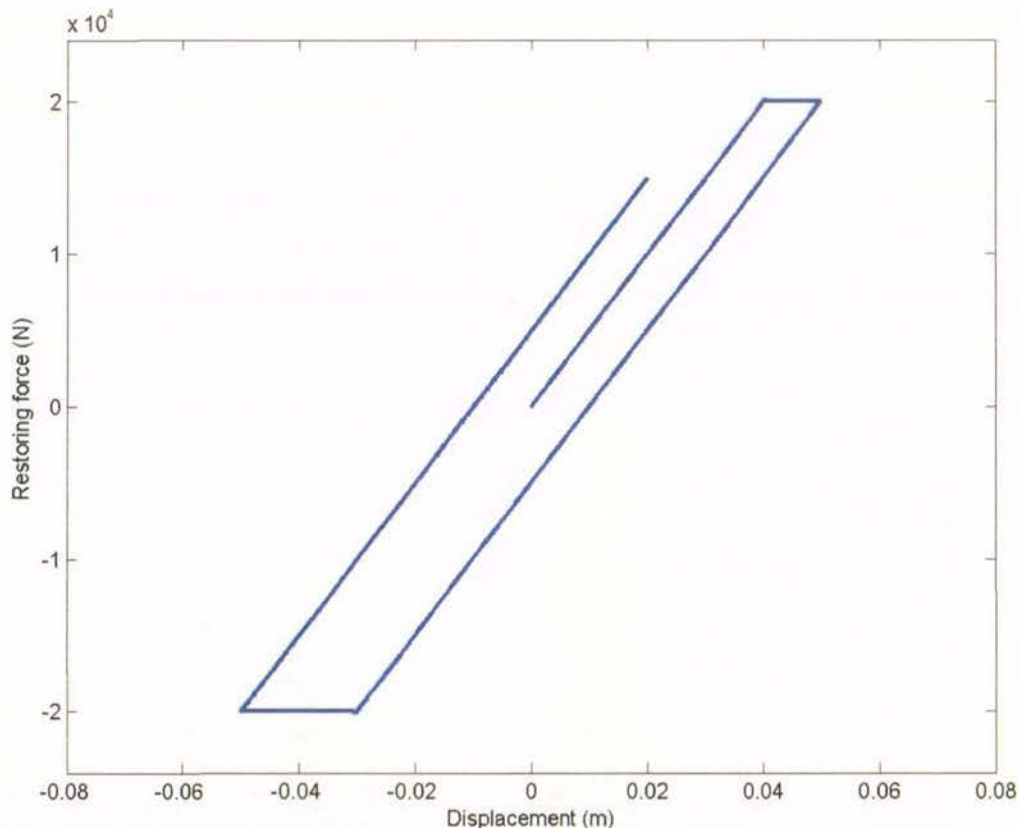


Figure 6.6. Force-displacement relationship for elastoplastic system.

A 1-DOF oscillator with mass of 1×10^4 kg, 5% damping and the above force-displacement relationship was subjected to Gaussian white noise excitation with a peak acceleration of 0.5g. A univariate AR(20) model was fitted to the accelerations with no noise added. Figure 6.7a shows the 1st AR coefficient identified using the forgetting factor approach. After the initial start-up phase the 1st AR coefficient shows sudden jumps correlated with the onset of yielding. Figure 6.7b shows when the structure was actually yielding, adopting the value of 1. The observed jumps in the 1st AR coefficient could be utilised to detect the on-set of yielding. By observing the magnitude of jumps corresponding to yielding onset, the following yielding indicator was proposed:

$$\text{detect} = |\theta_{1,t} - \theta_{1,t-1}| > 0.1 \quad (6.16)$$

where $||$ is the absolute value and $\theta_{1,t}$ and $\theta_{1,t-1}$ are the 1st AR coefficients at time steps t and $t-1$, respectively. It is proposed that if Equation (6.16) was true, yielding had occurred and the indicator assumed the value 1. Figure 6.7c shows the results of this yield indicator. Compared to Figure 6.7b, the proposed indicator of yielding has detected most of the yielding events in the time history.

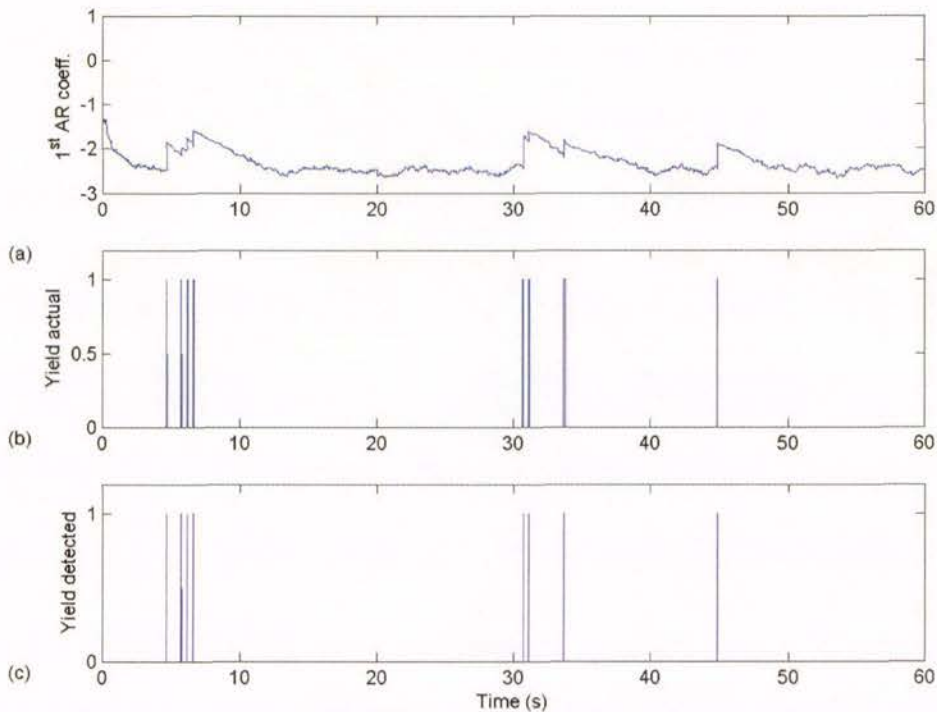


Figure 6.7. Damage detection in a 1-DOF elastoplastic oscillator: (a) 1st AR coefficient, (b) actual yielding, and (c) detected yielding.

6.4. Application to an analytical model of a 3-DOF nonlinear Bouc-Wen hysteretic oscillator

Previously, a simple 1-DOF oscillator with an idealised elastoplastic hysteretic restoring force, incorporating clearly defined elastic and plastic regions was investigated. Realistically, the transition between elastic and plastic behaviour would be smoother. The popular Bouc-Wen hysteretic model can be used to model a broad range of hysteretic restoring forces. The equations of motion for a 3-DOF system using a Bouc-Wen type hysteresis model can be written as follows

$$\begin{aligned} m_1 \ddot{x}_1 + r_1 - r_2 &= m_1 \ddot{u}_g \\ m_2 \ddot{x}_2 + r_2 - r_3 &= m_2 \ddot{u}_g \\ m_3 \ddot{x}_3 + r_3 &= m_3 \ddot{u}_g \end{aligned} \quad (6.17)$$

where x_i , $\dot{x}_i$ and $\ddot{x}_i$ are the displacements, velocities and accelerations of the i^{th} storey and $\ddot{u}_g$ is the ground excitation. The restoring forces for the 1st to 3rd storeys, r_1 , r_2 , and r_3 can be calculated from

$$\begin{aligned} r_1 &= c_1 \dot{x}_1 + \alpha k_1 x_1 + (1 - \alpha) R_{m1} z_1 \\ r_2 &= c_2 (\dot{x}_2 - \dot{x}_1) + \alpha k_2 (x_2 - x_1) + (1 - \alpha) R_{m2} z_2 \\ r_3 &= c_3 (\dot{x}_3 - \dot{x}_2) + \alpha k_3 (x_3 - x_2) + (1 - \alpha) R_{m3} z_3 \end{aligned} \quad (6.18)$$

where c_i are damping coefficients and R_{mi} are the yield restoring forces for the i^{th} storey. The parameter α is the ratio of post-yielding to pre-yielding stiffness. The fictitious hysteretic displacements z_i are found by solving the following nonlinear differential equations

$$\begin{aligned} \dot{z}_1 &= A \dot{x}_1 - \gamma |\dot{x}_1| |z_1|^{n-1} z_1 - \beta \dot{x}_1 |z_1|^n \\ \dot{z}_2 &= A \dot{x}_2 - \gamma |\dot{x}_2 - \dot{x}_1| |z_2|^{n-1} z_2 - \beta (\dot{x}_2 - \dot{x}_1) |z_2|^n \\ \dot{z}_3 &= A \dot{x}_3 - \gamma |\dot{x}_3 - \dot{x}_2| |z_3|^{n-1} z_3 - \beta (\dot{x}_3 - \dot{x}_2) |z_3|^n \end{aligned} \quad (6.19)$$

where A , γ , β and n are shape parameters. Equations (6.17)-(6.19) were solved using the 4th order Runge-Kutta method (Butcher 1987). This method solves a system of ordinary differential equations

$$\dot{\mathbf{y}} = f(t, \mathbf{y}) \quad (6.20)$$

with initial conditions

$$\mathbf{y}(t_0) = \mathbf{y}_0 \quad (6.21)$$

by iterative application of the following equations

$$\begin{aligned} \mathbf{k}_1 &= hf(t_i, \mathbf{y}_i) \\ \mathbf{k}_2 &= hf\left(t_i + \frac{h}{2}, \mathbf{y}_i + \frac{1}{2}\mathbf{k}_1\right) \\ \mathbf{k}_3 &= hf\left(t_i + \frac{h}{2}, \mathbf{y}_i + \frac{1}{2}\mathbf{k}_2\right) \\ \mathbf{k}_4 &= hf(t_{i+1}, \mathbf{y}_i + \mathbf{k}_3) \\ \mathbf{y}_{i+1} &= \mathbf{y}_i + \frac{1}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4) \end{aligned} \quad (6.22)$$

where h is the time step and i is the iteration step. In this case the system of equations was as follows

$$\dot{\mathbf{y}} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \\ \dot{y}_5 \\ \dot{y}_6 \\ \dot{y}_7 \\ \dot{y}_8 \\ \dot{y}_9 \end{bmatrix} = \begin{bmatrix} y_2 \\ (-m_1 \ddot{u}_g - r_1 + r_2) / m_1 \\ Ay_2 - \gamma |y_2| |y_3|^{n-1} y_3 - \beta y_2 |y_3|^n \\ y_5 \\ (-m_2 \ddot{u}_g - r_2 + r_3) / m_2 \\ A(y_5 - y_2) - \gamma |y_5 - y_2| |y_6|^{n-1} y_6 - \beta (y_5 - y_2) |y_3|^n \\ y_7 \\ (-m_3 \ddot{u}_g - r_3) / m_3 \\ A(y_8 - y_5) - \gamma |y_8 - y_5| |y_9|^{n-1} y_9 - \beta (y_8 - y_5) |y_9|^n \end{bmatrix} \quad (6.23)$$

where $\mathbf{y} = [x_1 \ \dot{x}_1 \ z_1 \ x_2 \ \dot{x}_2 \ z_2 \ x_3 \ \dot{x}_3 \ z_3]^T$.

For the 3-DOF model the initial tangent lateral stiffness of each storey was set to $k_1 = k_2 = k_3 = 1 \times 10^7$ N/m, lumped storey masses were set to $m_1 = m_2 = m_3 = 1 \times 10^4$ kg and the damping coefficients $c_1 = c_2 = c_3 = 1 \times 10^4$ Ns/m. The maximum restoring forces $R_{m1} = 1.2 \times 10^5$ N, $R_{m2} = 1.0 \times 10^5$ N and $R_{m3} = 1.0 \times 10^5$ N were adopted. These maximum forces were chosen to

prevent yielding being localised to the 1st storey, however, yielding at the 3rd storey was not explicitly sought. The following Bouc-Wen parameters were adopted $A = k_1/R_{m1}$, $\gamma = 0.5 \times k_1/R_{m1}$, $\beta = 0.5 \times k_1/R_{m1}$ and $n = 20$. The structure was excited with Gaussian white noise ground motion with a peak acceleration of 0.5g. Figure 6.7 shows the hysteretic restoring forces against the interstorey displacement. The 1st and 2nd stories show a degree of yielding and ductilities of 2.2 and 2.0 were reached respectively. The 3rd storey remained elastic.

A univariate AR(20) model was identified from the interstorey acceleration of each storey using the forgetting factor approach with $\lambda = 0.99$ adopted for the 1st and 2nd storeys and $\lambda = 0.995$ for the 3rd storey, respectively. Note, previously all AR models have been based on total or absolute accelerations. In the current investigation, the use of interstorey accelerations appeared to give better results. No noise was added to the accelerations. Figure 6.9 shows the 1st AR coefficient for all three models. In the figure, the value of all three coefficients for different storeys appeared to jump at various time instants and these jumps sometimes appeared to be simultaneous. This could indicate a departure from linear elastic behaviour.

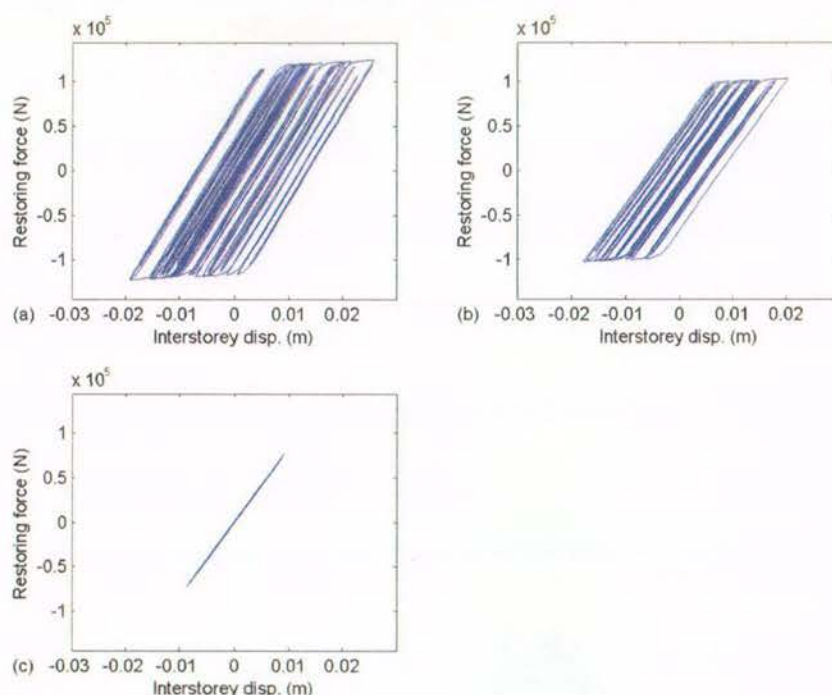


Figure 6.8. Hysteresis loops for 3-DOF Bouc-Wen structure under Gaussian white noise ground excitation: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

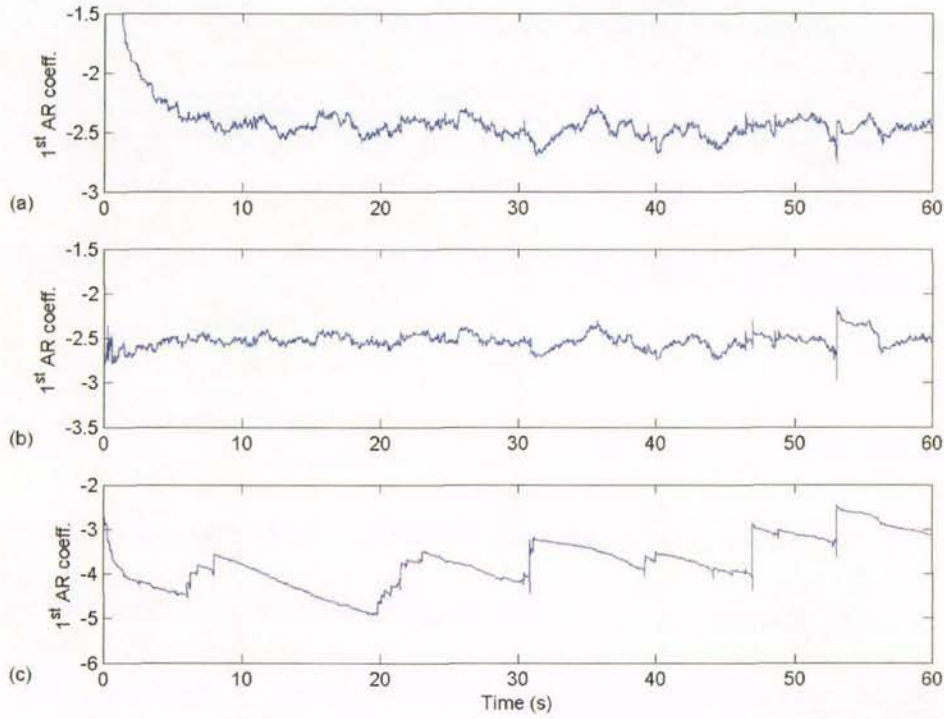


Figure 6.9. 1st AR coefficient for: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

Due to the smooth transition between elastic and plastic behaviour, the structure was assumed to sustain damage when the restoring force was greater than 99% of the maximum restoring force. Figures 6.10a and 6.10b show respectively the time instances at which the 1st and 2nd storeys were actually yielding, assuming the value 1 when yielding occurred. For detecting yielding events three indicators based on the AR coefficients were proposed

$$\begin{aligned}
 \text{detect } 1 &= \left| \theta_{1,t}^{(1)} - \theta_{1,t-1}^{(1)} \right| > 0.1 \\
 \text{detect } 2 &= \left| \theta_{1,t}^{(2)} - \theta_{1,t-1}^{(2)} \right| > 0.1 \\
 \text{detect } 3 &= \left| \theta_{1,t}^{(3)} - \theta_{1,t-1}^{(3)} \right| > 0.2
 \end{aligned} \tag{6.24}$$

where $\theta_{1,t}^{(1)}$, $\theta_{1,t}^{(2)}$ and $\theta_{1,t}^{(3)}$ are the 1st AR coefficients for the 1st, 2nd and 3rd stories at time step t , respectively. The indices assumed the discrete value 1 if Equation (6.24) was true, otherwise the value 0. All three indices are plotted in Figure 6.10c-e. Apart for initial false detections due to the initialisation of the AR models, all three indicators were able to correctly detect several yielding events. The first and second indicators detected the same yielding events. Figure 6.11a and 6.11b show that around these time instances there was a large yield in both stories in which the structure moved to a new centre of oscillation. Later,

the structure moved back and this event was also identified. The indicator based on the 3rd storey detected a larger number of events in both the 1st and 2nd stories.

To assess the effect of measurement noise on the approach, 1% Gaussian white noise was added to the analytical time histories. Using 2% or larger noise resulted in too many errors for consistent damage detection. Figure 6.12 shows the results in the same format as previously introduced using a forgetting factor of $\lambda = 0.988$ for all three storeys. Several yielding events were successfully detected by the three yield indicators and no false positive events were indicated apart from those caused by the initial start up phase. However, overall the performance of the approach was diminished. The three indicators appeared to detect mainly different events and only twice was a yield simultaneously detected.

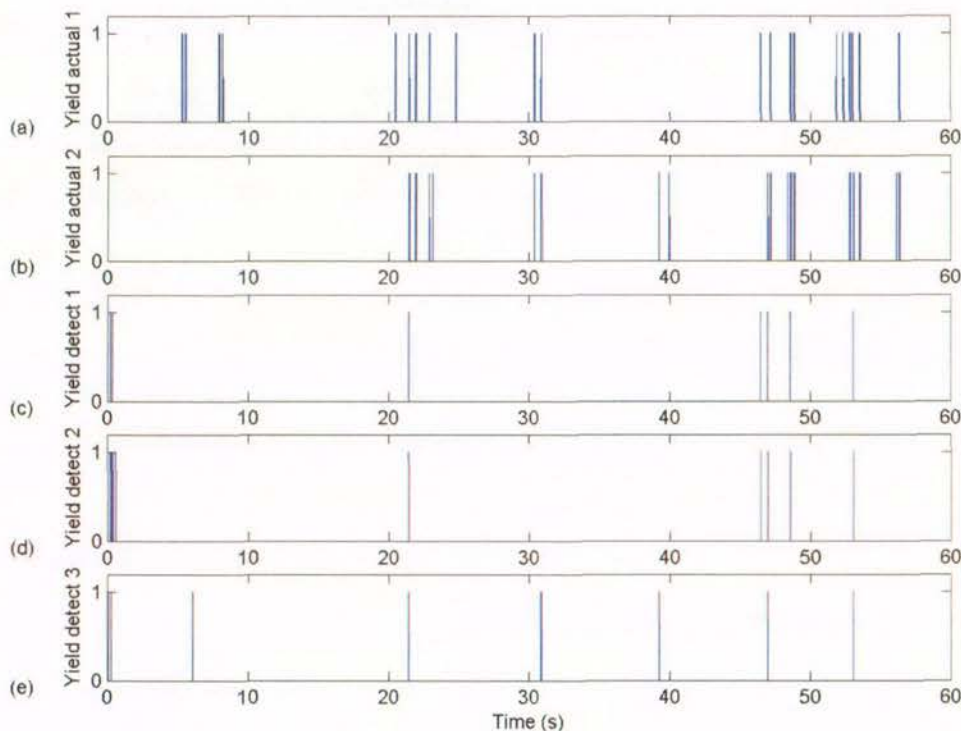


Figure 6.10. Detection of yielding in a 3-DOF Bouc-Wen structure: (a) actual 1st storey yield, (b) actual 2nd storey yield, (c) detected yielding events using detect 1 indicator, (d) detected yielding events using detect 2 indicator, and (e) detected yielding events using detect 3 indicator.

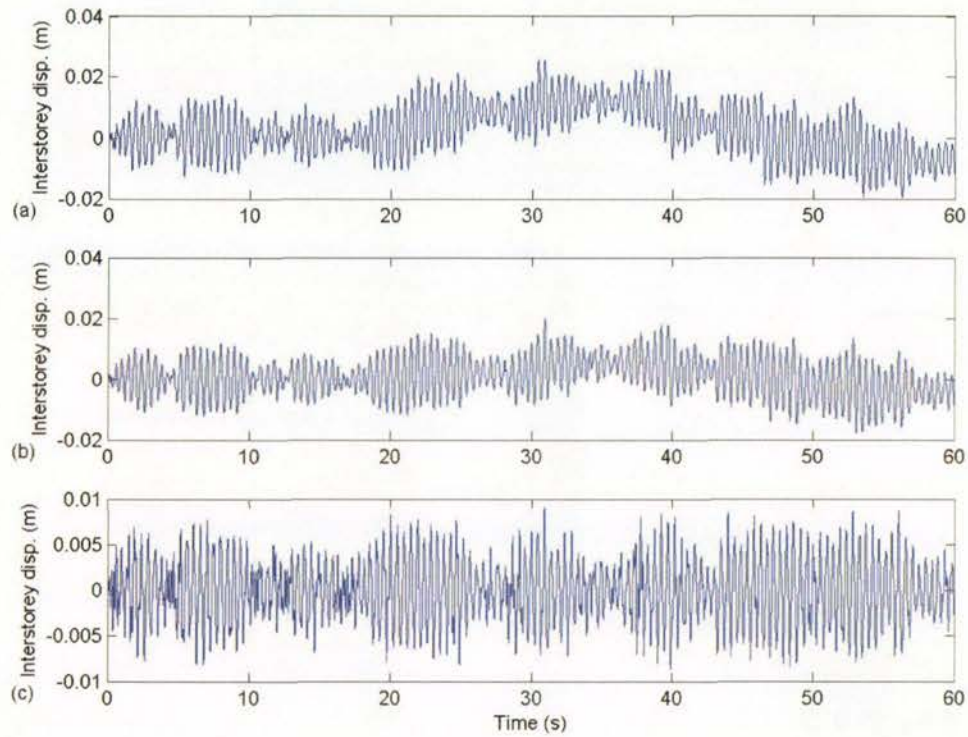


Figure 6.11. Interstorey displacements: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

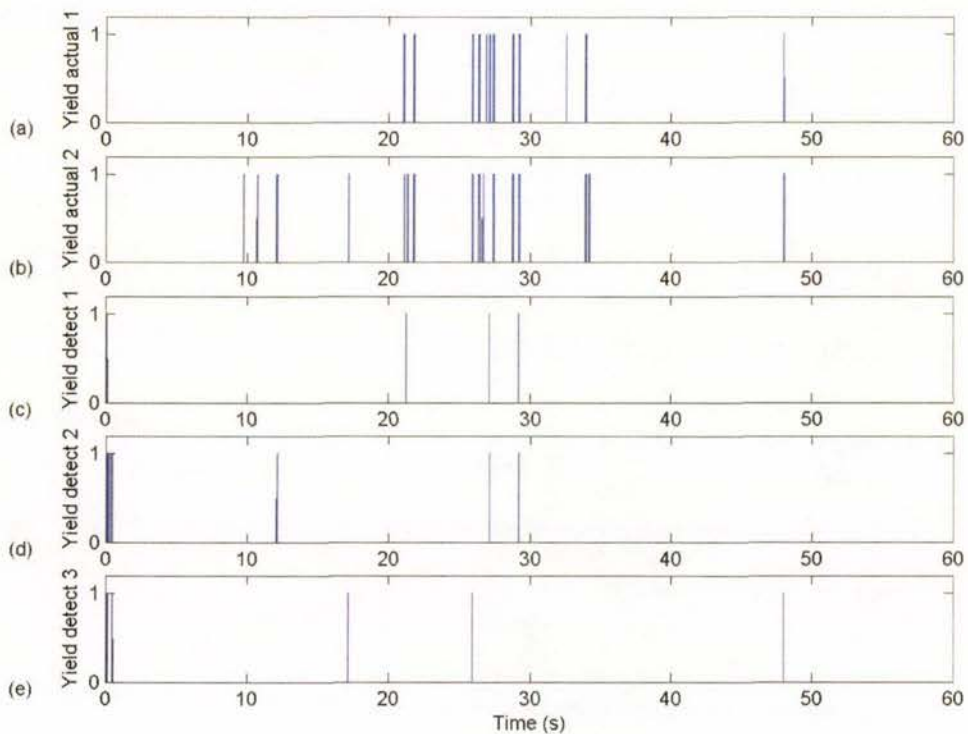


Figure 6.12. Detection of yielding in a 3-DOF Bouc-Wen structure with 1% noise: (a) actual 1st storey yield, (b) actual 2nd storey yield, (c) detected yielding events using detect 1 indicator, (d) detected yielding events using detect 2 indicator, and (e) detected yielding events using detect 3 indicator.

6.5. Conclusions

In this chapter, an online method of damage detection using recursive identification of the AR time series models has been presented. Firstly, a linear 3-DOF lumped-mass oscillator with damage introduced as sudden stiffness loss was studied. Two recursive system identification algorithms, the forgetting factor and Kalman filter, were investigated for AR coefficient identification. A BP ANN was trained to relate changes in the AR coefficients to stiffness and was used to track time dependent stiffness. The forgetting factor approach allowed for easier adjustment of the tracking properties than the Kalman filter and showed better results. Addition of 2% or 5% Gaussian white noise to the analytical time histories, simulating the effects of measurement noise, showed that good damage estimates and tracking were still obtainable.

Detection of nonlinear response on-set was investigated on analytical models of a 1-DOF elastoplastic oscillator and 3-DOF Bouc-Wen hysteretic system. The onset of nonlinearity manifested itself by distinctive, sudden jumps in AR coefficient values, and its detection was based on comparing the magnitudes of these jumps to preselected thresholds. Detection of yielding events in analytical time histories was generally successful. With the addition of 1% Gaussian white noise, many yielding events were still identified despite an overall reduction in performance and the number of events detected.

6.6. References

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CHAPTER 7

CONCLUSIONS

In this report, the techniques of statistical pattern recognition were applied to detect and assess the seismic damage in civil infrastructure. Methods were presented for damage detection from analysis of the dynamic responses. This chapter summarises the results and discusses directions for future research.

A damage detection methodology using time series analysis was proposed and studied. Initially, an offline damage detection method was developed, suitable for intermittent damage prognosis. Three experimental structures were studied, which presented varying degrees of complexity and damage mechanisms: a 3-storey bookshelf structure, the ASCE Phase II SHM Experimental Benchmark Structure and a RC column. The accelerations of the three structures were fitted using AR models whilst the structures were in undamaged and several damaged states. The coefficients of these AR models were chosen as damage sensitive features. The techniques of statistical pattern recognition: BP ANNs, NNC, LVQ and SOM were systematically applied for damage detection and quantification.

BP ANNs were used to recognise changes in the patterns in the AR coefficients and relate these changes to either a specific damage state (damage classification) or a reduction in structural stiffness (damage quantification). BP ANN demonstrated a very good performance in both tasks. Supervised learning techniques of NNC and LVQ were used to classify damage into states by assigning damage sensitive features to the reference feature clusters corresponding to known damage states. The ASCE Phase II SHM Experimental Benchmark Structure had a multi-sensor arrangement and the problem of feature dimensionality was addressed by either selection of a subset of accelerometers and/or AR coefficients or dimensionality reduction via PCA. PCA gave a more methodical approach and slightly better

results. The effect of changing operating conditions on the method was simulated by the addition of extra mass to the RC column. The techniques of NNC and LVQ were shown in some instances to be more effective classifiers than BP ANN. The results showed that successful damage classification was possible even when the data dimensionality was significantly reduced. An attempt was also made to quantify damage by observing the distance between a damage feature and centroids of clusters corresponding to known damage extent and results were promising.

The visualisation of AR coefficient data using two-dimensional projections using PCA and Sammon mapping showed some organisation of damage states into clusters. This was particularly present in the data from the ASCE Phase II SHM Experimental Benchmark Structure in which distinct clusters corresponding to various damage configurations could be seen. Unsupervised classification using SOM was attempted on data projections obtained from PCA and Sammon mapping. Surprisingly good results, comparable to NNC and LVQ, were obtained for the ASCE Phase II SHM Experimental Benchmark Structure, however, the results for the other two structures were poor.

The damage detection based on AR coefficients and pattern recognition was adapted to online damage detection using recursive techniques to identify the AR models online. Analytical studies on a 3-DOF linear building model showed that the forgetting factor and Kalman filter approaches in conjunction with a BP ANN were capable of detecting and tracking the extent of damage over time. The forgetting factor approach was preferred and showed good performance even in the presence of simulated Gaussian measurement noise.

Nonlinear online damage detection was investigated on a simple analytical model of a 1-DOF elastoplastic oscillator. The presence of nonlinearity could be identified online by observing abrupt changes in the AR coefficients. The approach was subsequently applied to an analytical model of a 3-DOF Bouc-Wen hysteretic system and good results were also achieved.

Future research may attempt to quantify damage using techniques other than ANNs. Supervised classification techniques have been shown to be effective, however, limited success has been achieved so far using unsupervised techniques. Such unsupervised methods would be useful to detect a departure from an initial, healthy state. For real structures

acquiring data from damaged states will be impractical and observing departures from a healthy state may be the only practical approach.

Linear time series methods have been known for some time and used exclusively in this research. However, although applications of nonlinear time series methods are in their infancy, they may prove useful. The characteristics of the embedding dimension, correlation dimension or information entropy could be incorporated with either BP ANNs, NNC, LVQ or SOM for damage detection in nonlinear systems.

More research needs to focus on applying techniques already developed to real-world structures. With the expanding instrumentation of buildings in New Zealand under the GeoNet monitoring system, data on real-world structures should become available for research. Applying the techniques developed in this research to such data would be an exciting and purposeful study that would realize a latent potential of GeoNet data.