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General Equilibrium Modelling - Construction and economic model for natural hazard management

Mark Walton, Phil Bishop, NZIER

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General Equilibrium Modelling

Constructing an economic model for natural hazard management

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Preface

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Authorship

This report has been prepared at NZIER by Mark Walton and Phil Bishop.

8 Halswell St, Thorndon
P O Box 3479, Wellington
Tel: +64 4 472 1880
Fax: +64 4 472 1211

Level 1, 77 Parnell Road
Parnell, Auckland
Tel: +64 9 309 0140
Fax: +64 9 309 0144

econ@nzier.org.nz
www.nzier.org.nz

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1. Introduction

1.1 Home page

Welcome to our practical CGE modelling website. The purpose of this website is to introduce analysts to Computable General Equilibrium (CGE) modelling as a means of assessing the economic consequences of naturally occurring catastrophic events, especially earthquakes. While our focus is on earthquakes, CGE modelling techniques can also be applied to the economic analysis of other natural disasters and consequent hazards such as volcanoes, floods, fires, tsunami inundation, and landslides. This website would also be of value to those just beginning to learn about CGE modelling, whether or not they have an interest in natural disasters.

2. Introduction Page

2.1 About our CGE modelling website

Welcome to our practical CGE modelling website. The purpose of these pages is to introduce analysts to Computable General Equilibrium (CGE) modelling as a means of assessing the economic consequences of naturally occurring catastrophic events, especially earthquakes. While our focus is on earthquakes, CGE modelling techniques can also be applied to the economic analysis of other natural disasters and consequent hazards such as volcanoes, floods, fires, tsunami inundation, and landslides.

In addition to estimation of the direct and indirect economic costs of natural disasters, a CGE model can be used to assist emergency management analysts with the development of disaster recovery strategies and the design of loss prevention and mitigation plans.

While the objective we have set for ourselves in preparing this information is perhaps a bit ambitious, the budget we have available to complete the task is not large. Hence the information presented here is necessarily of an introductory nature. Moreover, not all of the material is original. We draw on the work of others, much of which can be found elsewhere on the web. Our hope is that researchers interested in the economic analysis of the impact of natural disasters will find this material not only to be a useful starting point, but that it will take them well down the road of being able to undertake useful and careful analyses.

2.2 Why CGE models?

The quantitative analysis of the economic consequences of natural disasters has often been undertaken using variants of the input-output model. Examples of this approach include Cochrane (1974), Boisvert (1992), Rose et al. (1997), and Bishop et al. (2000). But there are significant limitations associated with input-output approaches for this type of analysis, as noted by these authors and others.

More recently, researchers involved in the economic analysis of natural disasters have begun to use CGE models. Chang et al. (2000) and Selcuk and Yeldan (2001) are two examples. A brief summary of the features of CGE models that make them a preferred framework for analysing natural hazard impacts and policy responses can be found in Chang et al. Key amongst these features is that CGE models explicitly incorporate markets and prices, and therefore permit the possibility of input substitution in the production process. Also, CGE models are highly amenable to the accommodation of engineering data or data based on informed judgment. This then makes the model a powerful tool for analysing a multitude of scenarios within in a consistent and unified framework.

The following questions typify the kinds of issues for which a CGE model can be used to help provide solutions:

1. What will be the impact in terms of GDP and output of a specific catastrophic event?
2. How will that impact be distributed across the sectors of the economy?
3. What will happen to prices in product and factor markets?
4. How will such price impacts affect the flow of goods and services into and out of the affected region, and for how long?
5. What are the optimal actions for the most rapid recovery of the physical capital loss, and in what sequence should they be undertaken?
6. What path will the economy take during the transition to a new long run equilibrium, following a catastrophic event such as an earthquake?

2.3 Intended audience

These web pages are aimed at the economic analyst who is not a CGE modeller, but wishes to understand the working and output of CGE models or wants to learn how to construct and use such a model. The material should be within the grasp of a senior undergraduate student in economics. There are a lot of CGE modelling resources available via the web, textbooks, and courses, but much of it is pitched at graduate student audiences. We view this website as an introduction to such material.

Analysts and researchers without formal qualifications in economics should also find this material both helpful and accessible. We would hasten to add, however, that a thorough understanding of the material contained here does not an economist make.

Being a computer programmer is not a prerequisite for learning about CGE models. However, a familiarity with computer processes and basic file management techniques would certainly be advantageous.

2.4 Sponsors

This project has been made possible through funding from two sources. The primary source of funding was a research grant, no. 6P01/481, from the New Zealand Earthquake Commission Research Foundation. The NZIER Public Good Research Fund also made a contribution.

It goes without saying that we are grateful to our funding sponsors. However, the total budget available to us to undertake this endeavour was less than required. As a consequence, we have been unable to prepare as much material, or go into as much detail, as we would have liked.

At some point in the future, and if we are able to secure the necessary funding, we will supplement the materials available from this website. In particular, we would like to add a section describing the microeconomic foundations of CGE models, and provide many more additional and well-documented templates of extensions to the basic model.

2.5 A note about presentation

At times throughout this website, we will be presenting snippets of GAMS code or even entire GAMS programs. In order to clearly distinguish it, the text of any such GAMS code will appear in a Courier font. The same font will also be used to distinguish commands that need to be entered at the computer's command line.

3. GAMS Page

3.1 Introduction

In order to construct and solve a CGE model on a computer, it is necessary to use a software package capable of doing so. We have chosen to use the GAMS software and in this section we explain how to obtain, install, and use GAMS.

GAMS, which stands for General Algebraic Modeling System, is designed to enable modellers to create and solve mathematical models of real world processes. It was originally created for use by economists, and although it is now used in many disciplines, it remains widely used by economists. A key strength of GAMS is that it focuses the user's attention on modelling issues, e.g. getting the economic structure right, and relieves the user from the technical burden of having to worry about things like memory management and designing solution algorithms.

There are other ways of formulating and solving CGE models. A suite of programs known as GEMPACK is also popular. Some modellers create their own algorithms to solve CGE models using programming languages such as C or FORTRAN, although we would not recommend this approach as it has a tendency to render the resulting model much less accessible to other interested researchers. Spreadsheet applications are not suitable for solving CGE models.

When we use the term CGE model, we are referring to "calibrated" CGE models. By far the majority of practical CGE models are of the calibrated variety. But it is possible, and indeed, not uncommon, to use statistical techniques to "estimate" all of the parameters of a CGE model. Such models, by virtue of data limitations, tend to be quite small (just 3 or 4 sectors) and used to analyse monetary policy or other macroeconomic topics. We are not concerned throughout this website with estimated CGE models, and one would ordinarily not use GAMS to solve such a model.

3.2 Installing GAMS

GAMS is a commercial piece of software. To use a licensed version, a license must be purchased. However, a free version can be downloaded from the GAMS website. The free, or unlicensed, version imposes restrictions on the size of the model that can be solved. All of the model templates that we present on this website will operate within these restrictions.

Instructions for installing the GAMS software are available from the GAMS website for a range of computer platforms. The installation process is quite straightforward and is similar to any other Windows based program. For the sake of clarity and brevity, we will assume throughout all of the material on this website that the computer platform being used is a PC with the Windows (95/98/NT/2000/XP) operating system.

3.3 Using GAMS

There are two ways of running GAMS once it has been installed.

The first is to use the Windows graphical interface that will be installed as part of the installation process - the GAMS developers call this the Integrated Development Environment or IDE. To run the IDE, you simply start GAMS from the start menu in the same way that any other application is started. We prefer not to use the IDE and are not very familiar with it. Nonetheless, there is no reason that users of the material to be found on this website could not use it if they wish. While operation of the IDE is not difficult, some notes to assist with learning how to use it have been prepared by Bruce McCarl.

The second way of using GAMS is to use a text editor in conjunction with running GAMS from a "dos" box. This is the method we prefer. By dos box we simply mean the window that allows you to issue instructions to the computer directly from the command line. For example, if you issue the command cmd in the run dialog window from the start menu, what you get is a dos box. Whereas the text editor is used to create and/or edit a GAMS program, the GAMS program itself is compiled and executed by issuing a command line instruction from within the dos box.

For example, if a GAMS program was created and saved as a text file called `mymodel.gms`, then that file would be processed (i.e. compiled and executed) by issuing the command `gams mymodel` in the dos box.

There are numerous text editors available and many of them are even free. Even though the IDE has in-built text editing capabilities, it is probably advisable to choose a text editor and get familiar with it, whether you use the IDE or not. The text editor that we use, which is not free, is called Epsilon.

Note that after saving your GAMS program from within the text editor, it is not necessary to close the file out of the editor before running the GAMS program from the dos box. All you need to do is simultaneously press the Alt and Tab keys to switch back and forth between the text editor and the dos box, all the while keeping your GAMS program visible in the text editor.

There are two other minor details that we would recommend taking care of at the time GAMS is installed.

Add the GAMS system directory to your path. Doing so is especially useful if you plan on running GAMS from a dos box. The GAMS system directory is the place where GAMS will be installed on your computer (probably in c:\Program Files\Gams). The GAMS installation notes explain how to add the GAMS system directory to your path; note that the process differs slightly depending on the version of Windows you have on your computer.

Make the solver called PATH the default solver for MCP models. This will ensure that when we start solving CGE models, GAMS will produce output that appears the same for you as it does for us. Associating default solvers with the various model types is explained in the installation notes. If your GAMS software is unlicensed, the choice of default solvers is probably not that important. Nevertheless, we suggest you choose PATH as the default MCP solver.

3.4 Learning GAMS

Learning the GAMS language syntax is not difficult - it just takes a bit of practice. The mechanics of actually using GAMS are not complicated either:

1. Write out a collection of GAMS statements using a text editor or the IDE, and save the result as a text file ending with the suffix gms (e.g. mymodel.gms).
2. Tell GAMS to run the program you have just created. If using the IDE you can press the F9 key. If using a dos box, then at the command prompt type gams mymodel and hit enter. This will cause GAMS to compile the program, check it for errors, and if there aren't any it will execute all of the statements in the program. Whether there are errors or not, an output file will be generated whose name is the same as the input file except that the suffix will be lst (e.g. mymodel.lst).
3. View the output file using a text editor or the IDE.

The best way to learn the GAMS language syntax and the mechanics of running GAMS programs is quite simply to practice. This means literally writing out the code yourself, rather than blindly using examples downloaded from the web or the GAMS model library. Depending on your aptitude for programming, it will only take an hour or two to acquire the basic skills. If you don't practice and make a few mistakes, it will take longer.

As a first exercise, we would strongly recommend carefully going through the GAMS tutorial prepared by Rick Rosenthal. If further learning assignments are required, there are two very good documents that can be found in the Contributed Documentation section of the GAMS website: Ron Rardin's Notes on GAMS for Optimization, and pp 121-159 of McKinney and Savitsky's GAMS Tutorials for Water and Energy Management.

Although this will quickly become apparent when working through the tutorial, we finish this section by offering a sneak preview of what constitutes a GAMS program. When stripped back to the bare essentials, there are really only six main elements to a GAMS program.

SETS - the indices upon which parameters, variables, and equations are defined. Sets can be viewed as the subscripts you use when writing out an algebraic expression.

DATA - the information you give to the model. In simple cases, data is entered using the Parameter, Table, and Scalar statements. Or it may be the result of an assignment statement, i.e. a GAMS statement that evaluates the right hand side of an expression, and assigns the result to the symbol (a parameter or a scalar) on the left hand side. In more sophisticated programs, the data may be read in from an external file such as a spreadsheet (although even then, the data will be allocated to a Parameter, Table, or Scalar symbol).

VARIABLES - these are the things whose value we want the model to tell us (obviously).

EQUATIONS - expressions made up of parameters and variables. "Equations" may be either strict equalities or inequalities.

MODEL and SOLVE - statements that tell GAMS which equations are to be used to comprise a model, and to solve the model. A program might contain more than one model.

REPORTING - reports or output can take several forms. It may be a simple table showing the level of one or more variables, which was the result of using the Display statement in GAMS. It may be tables showing the results of calculations performed after the model has been solved but which use the optimal level of one or more variables. In a more sophisticated model, detailed reports can be generated in external files using the Put facility or the GDX facility.

The GAMS syntax is free of rigid formatting requirements. It is case insensitive and there are no rules regarding indentation, spaces between lines, or even the number of statements that can be included in a single line. However, it is advisable to adopt a format that allows the code to be read easily by a human.

Finally, we would recommend that you don't store your work files, e.g. the .gms and .lst files, in the system directory. Whereas the location of the GAMS system files is called the system directory, the location of your work files, wherever that may be, is referred to as the working directory.

3.5 GAMS and MPSGE

Before we leave this section on using GAMS, we need to mention MPSGE. It is probably sensible, although by no means required, that users get comfortable with the basics of writing and running standard GAMS programs before moving on to MPSGE.

MPSGE, which stands for Mathematical Programming System for General Equilibrium, is a GAMS subsystem designed specifically for formulating and solving CGE models. It was created by Tom Rutherford.

Although the syntax for MPSGE is quite different to that of GAMS, the MPSGE code is written directly into a GAMS program. Hence the process for solving a model created using MPSGE is the same as that for solving a model created using GAMS, i.e. you save the code in a text file, run GAMS, and look at the resulting output in the .lst file.

Most of the model templates we present throughout this website will make use of MSPGE. The structure of the models and the syntax will be explained as the models are encountered, beginning with the simplest of formulations and then slowly adding complexity.

As will become clear when we get to the stage of using MSPGE, the models created using MPSGE are actually formulated as Mixed Complementarity Problems (MCPs). While MCPs will be explained when we get to them, an introductory exposition can be found here.

4. Input Output Page

4.1 The IO table

An input-output (IO) table is a matrix representation of the flows related to the production and consumption of goods and services for a particular economy. A stylised picture of an input-output table is presented in Figure 1.

Figure 1 The input-output table

	Industry inputs (j)				Final demands (f)				Total sales (x)
Industry sales (i)	z_{11}	z_{12}	…	z_{1n}	c_1	i_1	g_1	e_1	x_1
	z_{21}	z_{22} I	…	z_{2n}	c_2	i_2 II	g_2	e_2	x_2
		…				…			
	z_{n1}	z_{n2}	…	z_{nn}	c_n	i_n	g_n	e_n	x_n
Value added (v)	l_1	l_2 III	…	l_n		IV			l
	k_1	k_2	…	k_n					k
Imports (m)	m_1	m_2	…	m_n		m_f			m
Total outlays (x)	x_1	x_2	…	x_n	c	i	g	e	

The first *n* rows of Figure 1 record the distribution of a sector's output; the first *n* columns record the distribution of a sector's inputs. The cells in the intersection of the first *n* rows and columns represent the interindustry transactions; this square array of transactions is known as the transactions matrix, or quadrant I of the input-output table, and is denoted **Z**.

Purchases by final consumers from each sector *i* are recorded in the array to the right of the transactions matrix (i.e. quadrant II). These purchases can be broken down into:

$c_i =$ personal consumption expenditure on goods from sector *i*

$i_i =$ purchases of fixed assets for investment on goods from sector *i*

$g_i =$ government purchases on goods from sector i, and

$e_i =$ exports of goods from sector i.

Sector j's payments to the factors of production are recorded in quadrant III (i.e. the array below the transactions matrix). In Figure 1 these are:

$l_j =$ payments made to labour

$k_j =$ payment made to owners of capital, including the depreciation allowance.

Also included within quadrant III are imports of intermediate inputs by sector j, m_j. Imports purchased by final consumers, m_f, are recorded in quadrant IV.

So, looking across a row i reveals the disposition of total gross output for industry i, whether it is used as an input to other industries (quadrant I), or is supplied to final demand (quadrant II). The gross output of an industry is given by the industry row sum. Looking down a column shows the total inputs into an industry, both from "primary" inputs (quadrant III) and from other industries (quadrant I). The industry row sums and their corresponding industry column sums are equal as the sum of all purchases must equate to the sum of all sales in the input-output framework. Note that returns to capital, or operating surplus, appear as a "primary" input. Quadrant IV is typically empty, except for imports directly into final demands.

4.2 Multipliers

Impact analysis is one form of analysis often undertaken within the input-output framework, and requires the calculation of multipliers. A basic tenet of the input-output model is that the various components of final demand (i.e. the columns in quadrant II – household consumption, exports, investment, etc.) are considered to be exogenous. Hence, the question one asks when calculating multipliers is: what is the impact on, say, output or income or employment following an exogenous increase in final demand for the output of the sector of interest? Such an increase might take the form of an increase in export demand. In other words, given a unit (say, $1 million) increase in final demand for industry j's output, what is the total direct and indirect impact once a new equilibrium has been attained? An output multiplier is simply the figure by which an initial change in output throughout the entire economy should be multiplied to calculate the total change in output resulting from that initial exogenous change (recall that final demand is an element of gross output).

Using the input-output framework, it is easy to see that for an industry to deliver an additional $1 million worth of final demand, that industry is required to directly increase its gross output by $1 million. But that is not the end of the story. To produce an additional $1 million of gross output in one sector, additional inputs to that sector are needed from other industries. And to produce the additional output required in these other industries, additional inputs are needed from yet other industries. And so it continues. This process is sometimes referred to as the initial and subsequent expenditure rounds. A new equilibrium is attained when the value of a new round of purchases does not change from the current round. The total additional output needed from *all* industries is the amount by which total gross output rises in response to an exogenous $1 million increase in final demand. (The exogenous increase is usually, although not necessarily, associated with a single sector).

While it is possible to compute multipliers for a wide range of economic variables, those of most interest are typically output, income, value added, employment, and import multipliers. Multipliers are computed on a sectoral basis. The output multiplier for the j^{th} sector is the total change in gross output (or sales) of the entire economy divided by the initial change in output in sector j, where the initial change is nothing more than the exogenous increase in final demand. Income multipliers are defined slightly differently and reflect the total income change in the economy per unit of income generated in the j^{th} sector, where the income change in the j^{th} sector is a direct result of the exogenous increase in final demand of the j^{th} sector.[1] Value added multipliers are similar to those for income, but refer to the increase in value added generated throughout the economy as a result of increased output in the j^{th} sector. Employment multipliers are similarly analogous to the income multipliers except that instead of income, they refer to the number of additional full-time equivalent employees utilised in the economy as a result of increased output in the j^{th} sector.

To make matters even more confusing, two "types" of each multiplier are calculated; the so-called type I and type II multipliers. Type I multipliers follow the intuition described above. They include the "direct" effect on output in the industry which experiences an exogenous increase in demand and the "indirect" effect resulting from the need for all other industries to produce more inputs for that industry. Type II multipliers include an additional effect, the so-called "induced-income" effect.

The induced income effect arises because as firms produce more output, households receive more income (i.e. workers receive wages, investors receive dividends, proprietors receive a return to their management skill, etc.), which they in turn spend on food, cars, holidays, TVs and a range of

[1] Two technical points of clarification are warranted at this juncture. First, the input-output model assumes a linear production technology and thus input requirements increase in proportion to output. Compensation of employees is one such input. Second, we have adjusted the data in the input-output table so that compensation of employees captures payments to the self-employed, proprietors income, and dividend payments, as well as wage and salary payments. It is this adjusted value, net of taxes, that we refer to as household income when we compute income multipliers.

other things. So total output in the industries that produce all these other goods also rises as final demand has increased. Hence, increased output means increased income for households which *induces* yet more consumption and therefore output, which creates additional income. Like the type I multiplier, the type II multiplier measures the impact at the point at which a new equilibrium is reached.

To illustrate, the employment multiplier for an industry is the total increase in employment in all industries divided by the initial increase in employment in the sector directly affected by a $1 million increase in demand for output. The type I multiplier includes the "direct" effect (which is the same as the initial impact) and the "indirect" effect, which is the flow-on to industries that supply the first industry with inputs. The type II multiplier includes both these effects plus the "induced income" effect caused by increased demand for outputs from all industries by households who now have a higher income.

Multipliers are easy to interpret incorrectly. It is tempting, for example, when a particular sector's income multiplier has a value 4 to interpret this as meaning that a $1 million increase in output in that sector causes a $4 million increase in income. This is incorrect. What an income multiplier of 4 means is that the final impact on income of a $1 million increase in output in that sector is four times the initial impact on income following that increase in output. The initial impact may be quite small and hence a small total impact can give rise to a large multiplier.

4.3 Derivation of multipliers

This section explains the use and derivation of IO multipliers. For convenience, the diagram of the structure of the IO table is repeated here.

Figure 2 The input-output table

	Industry inputs (j)				Final demands (f)				Total sales (x)
Industry sales	z_{11}	z_{12}	…	z_{1n}	c_1	i_1	g_1	e_1	x_1
(i)	z_{21}	z_{22} I	…	z_{2n}	c_2	i_2 II	g_2	e_2	x_2
		…				…			
	z_{n1}	z_{n2}	…	z_{nn}	c_n	i_n	g_n	e_n	x_n
Value added (v)	l_1	l_2 III	…	l_n		IV			l
	k_1	k_2	…	k_n					k

Imports (m)	m_1 m_2 ... m_n	m_f	m
Total outlays (x)	x_1 x_2 ... x_n	c i g e	

4.3.1 Type I and Type II multipliers

As noted above, Type I and Type II multipliers differ in the extent to which they fully capture economy-wide impacts of a sectoral change. Type II multipliers provide a more comprehensive measure of economic change. The derivation of Type II multipliers is essentially an extension of the Type I algebra; hence both Type I and Type II derivations are presented here.

The distinction between Type I and Type II multipliers is as follows:

- *Type I multipliers* measure the direct and indirect effects of a change. In the instance of an output multiplier, the direct effect is the initial rise in output in the industry which is experiencing higher demand. The indirect effects result from the need to produce more inputs for that industry.
- *Type II multipliers* include the direct and indirect effects, as well as the income-induced effect of a change. The initial direct and indirect effects result in higher employment, which in turn boosts household income, which increases demand, which lifts output, which then lifts employment further, and so on.

4.3.2 Derivation of Type I multipliers

Given an n-sector economy, the transactions matrix and the vectors of final demands and outputs can be represented as:[2]

$$\mathbf{Z}=\left(\begin{array}{cccc|c} z_{11} & z_{12} & \cdots & z_{1n} & z_{1c} \\ z_{21} & z_{22} & \cdots & z_{2n} & z_{2c} \\ \vdots & \vdots & & \vdots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nn} & z_{nc} \\ \hline z_{c1} & z_{c2} & \cdots & z_{cn} & \end{array}\right) \qquad \mathbf{f}=\begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix} \qquad \mathbf{x}=\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

where:

z_{ij} = sector i sales to sector j

f_j = sector j sales to final demand

[2] In the Inter-industry Study 1996, which forms the basis of the multiplier analysis contained in this report, n = 126.

x_j = total sector j sales

The c-th row represents compensation of employees (i.e. payments for labour), and the c-th column is household consumption.

The relationship between the elements of these matrices is:

$$x_i = z_{i1} + z_{i2} + \ldots + z_{in} + f_i \quad (1)$$

The technical coefficients (or direct input coefficients) of sector j are written:

$$a_{ij} = z_{ij} / x_j \quad (2)$$

which in matrix form is:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

Thus a_{ij} is the proportion of sector j's total output (the value of which is equivalent to the value of sector j's total input) and is made up of inputs from other sectors.

Given equation (1), sector i's sales can be rewritten and expressed in terms of technical coefficients as:

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n + f_i \quad (3)$$

Equations (1) and (3) respectively can be written in matrix form as:

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f} \quad (4)$$

$$\mathbf{x} = \mathbf{Ax} + \mathbf{f} \quad (5)$$

where **i** is an n-element column vector of 1s.

Recall that equations (1) and (3), and hence (4) and (5), are equivalent.

Using an nxn identity matrix, **I**, and rearranging equation (5) yields:

$$\mathbf{Ix} - \mathbf{Ax} = \mathbf{f}$$

$$\Rightarrow \quad (\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{f} \quad (6)$$

From this we can derive the change in output, $\mathbf{x}^*$, arising from a change in final demand, $\mathbf{f}^*$:

$$\mathbf{x}^* = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}^* \quad (7)$$

$(\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief Inverse, or the total (initial, direct and indirect) requirements matrix. This can be represented by **B** so that:

$$\mathbf{x}^* = \mathbf{B}\mathbf{f}^* \quad (8)$$

a) Output multipliers

Re-expressing equation (8) in expanded format gives:

$$\mathbf{x}^* = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix} \begin{pmatrix} f_1^* \\ f_2^* \\ \vdots \\ f_n^* \end{pmatrix}$$

From this it can be seen that the economy-wide impact of f_j^* is:

$$x_j^* = \sum_{i=1}^{n} b_{ij} f_j^* \quad (9)$$

For $f_j^* = 1$, x_j^* reduces to:

$$x_j^* = \sum_{i=1}^{n} b_{ij} \quad (10)$$

x_j^* is the (Type I) *output multiplier*: that is, how much does economy-wide output have to increase to meet a $1 increase in final demand for the output of sector *j*.

b) Value added multipliers

In principle these are calculated in the same way as for output multipliers; the distinction is that changes in sectoral output arising from a change in final demand are scaled by each sector's value added input coefficient (i.e. the ratio of value added to total inputs).

The value added input coefficients are calculated using the sum of the compensation of employees, the operating surplus and net indirect tax rows of the input-output table. We shall denote this sum as z_{vaj}. In a manner similar to that used to derive the direct input coefficients in equation (2), the value added input coefficients are:

$$a_{vaj} = z_{vaj} / x_j \tag{11}$$

By using this to scale the impact of changes in output we have:

$$v_j^* = \sum_{i=1}^{n} a_{vai} b_{ij} / a_{vaj} \tag{12}$$

This is the (Type I) *value added multiplier*. Its interpretation is: how much will economy-wide value added increase, above the initial increase in sector j's value added payments, given an increase in final demand of sector j's output of \$1.

c) Income multipliers

These are calculated as for the value added multipliers, with scaling of sectoral output done by compensation of employees (i.e. household income). Recalling that compensation of employees is recorded in c-th row of the input-output table, the compensation of employees coefficient is:

$$a_{cj} = z_{cj} / x_j \tag{13}$$

By using this to scale the impact of changes in output we have:

$$h_j^* = \sum_{i=1}^{n} a_{ci} b_{ij} / a_{cj} \tag{14}$$

This is the (Type I) *income multiplier*. Its interpretation is: how much will economy-wide income increase, above the initial increase in sector j's income (i.e. compensation of employees) payments, given an increase in final demand of sector j's output of \$1.

d) Employment multipliers

These are calculated as for the income multipliers, but rather than use compensation of employees to scale the output effects we have used the ratio of full-time equivalent (FTE) jobs to output by sector. This employment ratio is:

$$e_j = FTE_j / x_j \tag{15}$$

Using this in our multiplier calculation gives:

$$e_j^* = \sum_{i=1}^{n} e_i b_{ij} / e_j \tag{16}$$

4.3.3 Derivation of Type II multipliers

In the calculations above, the matrix elements are restricted to those within the *n*x*n* confines of the transactions matrix of the inter-industry table. However, this effectively excludes the impact of changes in household income arising from additional final demand, since household income and consumption is outside of the *nxn* matrix. Type II multipliers address this issue by expanding the *nxn* matrix to include household consumption and compensation of employees. Households are effectively treated as another production sector in Type II multiplier analysis, producing labour services and demanding consumption goods and services.

The technical coefficients for the household row and column are:

$$a_{cj} = z_{cj} / x_j \qquad (17)$$

$$a_{ic} = z_{ic} / x_c \qquad (18)$$

where:

a_{cj} = the labour coefficient for sector j

a_{ic} = the 'household consumption' coefficient.

In equation (18), x_C represents household disposable income. For the analysis contained in this report we calculated household disposable income as the sum of:

compensation of employees (from the input-output tables)

self-employed earnings (derived from SNZ's *Institutional Sector Accounts*)

dividend earnings (derived from SNZ's *Institutional Sector Accounts*)

and then subtracted tax from that sum using an average personal income tax rate derived from the *Institutional Sector Accounts*. Note that both self-employed earnings and dividend earnings are reflected in the operating surplus row of the input-output table.

5. The SAM Page

5.1 What is a SAM?

A social accounting matrix is a means of portraying specific details on various economic flows. It consists of rows and columns that represent the payments (or inputs) and receipts (or outputs) of specific institutional units. In a simple SAM these units are likely to include producers of goods and services, households, government and the rest of the world. The level of detail embodied in the SAM is dependent firstly on the purpose for which the SAM is to be used, and secondly (but no less importantly) on the extent to which suitable data is available.

A simplified representation of a New Zealand SAM is shown in Figure 3. As can be noted, the SAM embodies both a functional distinction – separating, for example, production of goods and services from the consumption or investment of those same products – and an institutional distinction, where units that face similar decision making rules may be grouped together. In Figure 3, these "institutional sectors" are producer enterprises, households, government and the rest of the world.

In a SAM, column entries typically represent inputs, payments or outlays, while rows represent outputs, receipts or incomes. Thus the household column represents payments by households, while its corresponding row represents household income. The intersection of one sector's row with another sector's column represents a transfer of some kind, which may or may not be requited, between the sectors.

SAMs have traditionally provided the basis for analysing the impact of government policy in developing economies, either as a stand alone tool, or as the basis for economic models. Institutions such as the World Bank and the International Food Policy Research Institute make extensive use of social accounting matrices in their computable general equilibrium (CGE) modelling and analysis.

As CGE models have become the pre-eminent tool for analysing a range of government policy scenarios, and exogenous shocks to an otherwise stable economy, so have SAMs become an increasingly important device for providing the basis on which such models are built.[3]

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[3] See, for example, J F Francois and K A Reinert (1997) for an account of the role SAMs and CGE models play in trade policy analysis.

Figure 3 An aggregated SAM

	Commodities	Industries	Factors	Producer enterprises	Households	Government	Investment	Rest of the World
Commodities		Intermediate consumption			Household consumption expenditure	Government consumption expenditure	Gross fixed capital formation	Exports
Industries	Gross output							
Factors		Value added						
Producer enterprises			Operating surplus					
Households			Compensation of employees	Primary income		Current transfers		Foreign remittances
Government	Product taxes	Other indirect taxes		Enterprise taxes	Personal taxes			
Investment				Retained earnings	Household savings	Government savings		Capital transfers from abroad
Rest of the World	Imports				Current transfers abroad	Current transfers abroad	Capital transfers abroad	

Source: NZIER

5.2 Previous work

There has been little SAM-related work with a New Zealand focus to date. Notable examples include:

- Brian Philpott's 1990 work for the Research Project on Economic Planning.[1] This was a reasonably highly aggregated demand-based SAM, used to gauge the impact of European Community integration on the New Zealand economy, and to make medium term projections of the Asian Industrialising Region.

- Infometrics' ESSAM (Energy Substitution, Social Accounting Matrix) model.[2] This could fairly be described as a "partial" SAM, portraying in some detail the inter-linkages between industries, but containing a limited amount of information regarding the connection between production and the various institutional sectors.

- The NZIER SAM.[3] The NZIER social accounting matrix has evolved over time into the SAM described here.

[1] Philpott, B, 1992.

[2] As used, for example, in Infometrics Consulting, 2000.

[3] As used, for example, in Bishop, P, et al, 2002.

5.3 The SAM and the National Accounts[4]

The purpose of this section is to provide an overview of the sequence of the national accounts, since a social accounting matrix has a very strong foundation in that sequence.

A quote from the *System of National Accounts 1993* itself probably best encapsulates the nature of the System:

> The System of National Accounts (SNA) consists of a coherent, consistent and integrated set of macroeconomic accounts, balance sheets and tables. It provides a comprehensive accounting framework within which economic data can be compiled and presented in a format that is designed for purposes of economic analysis, decision-taking and policy making.
>
> The System is built around a sequence of interconnected flow accounts linked to different types of economic activity taking place within a given period of time, together with balance sheets that record the values of the stocks of assets and liabilities held by institutional units or sectors at the beginning and end of the period. Each flow account relates to a particular kind of activity such as production, or the generation, distribution, redistribution or use of income. Each account is balanced by introducing a balancing item defined residually as the difference between the total resources and uses recorded on the two sides of the account. The balancing item from one account is carried forward as the first item in the following account, thereby making the sequence of accounts an articulated whole. The balancing items typically encapsulate the net result of the activities covered by the accounts in question and are therefore economic constructs of considerable interest and analytical significance - for example, value added, disposable income and saving. There is also a strong link between the flow accounts and the balance sheets, as all the changes occurring over time that affect the assets or liabilities held by institutional units or sectors are systematically recorded in one or another of the flow accounts. The closing balance sheet is fully determined

[4] This section draws heavily from *System of National Accounts 1993*. All terminology and corresponding definitions are drawn from this reference, which sets out, in considerably more detail, the workings of the sequence of national accounts.

> by the opening balance sheet and the transactions or other flows recorded in the sequence of accounts.[5]

Thus the emphasis is on the flows – or transactions – that are related either to production, or to the distribution and redistribution of income amongst institutional units.

Traditionally, the SNA is presented in a T-account format (or variation thereof). A social accounting matrix, on the other hand, is essentially a matrix representation of the same information, with one important extension: a SAM allows more detailed inter-sectoral flows to be displayed than can be done in the traditional T-account format. Although the traditional format shows, for example, how much interest each institutional sector has paid and received, it does not show the counterpart to each transaction. As noted (see Figure 3) the SAM format records the payments by each institutional unit in its columns, and the receipts by those units in its rows. Thus, the intersection of a row and column represents a transaction between the corresponding institutional units, a level of detail which is not able to be shown in the T-account format.

SAMs can easily be aggregated to the level most appropriate for the modelling question at hand, one characteristic which makes them a useful part of CGE analysis. At their most detailed SAMs can contain information regarding individual flows between institutional units – for instance, the payment of income tax by salary earners (households) to government. At a more aggregated level, transactions can be aggregated within institutional sectors; an example of the layout of such a SAM is shown in Figure 3.

5.4 SAM content

In simple terms (and as noted above), the column entries of a SAM generally represent inputs, expenses or payments, while row entries typically represent outputs, income items or receipts. This section outlines the types of flows that are recorded in a SAM, as shown in Figure 3. The column and row headings are used as a guide.

5.4.1 Commodity supply/use

The column headed "Commodities" records the supply of goods and services to the New Zealand economy. This comprises:

• Domestic supply: shown broken down by supplying industry, in the intersection of the "Commodities" column and "Industries" row.

• Imported goods and services: shown in the intersection of "Commodities" and "Rest of the World".

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[5] *System of National Accounts 1993,* p.1

• Taxes levied on the sale of goods and services (shown in the intersection of the "Commodities" and "Government" row. The most obvious example is GST.

The row headed "Commodities", on the other hand, records the use of all commodities by the New Zealand economy. This comprises:

• Intermediate consumption: that is, the use of goods and services by producers as intermediate inputs; shown in the intersection with the "Industries" column.

• Household consumption expenditure: shown in the intersection with the "Households" column. This shows the use by households of goods and services.

• Government consumption expenditure: shown in the intersection with the "Government" column.

• Gross fixed capital formation: recorded in the intersection with the "Investment" column, this shows the use of goods as fixed assets in the production process.

• Exports, shown in the "Rest of the World" column.

5.4.2 Industry input/output

The "Industries" row and column, show the various inputs to, and outputs of, the production process, respectively. The intersections with commodity supply and use are described above. The remaining entries – that is, the intersection between the "Industries" column and the "Factors" and "Government" rows – represents the use of the so-called "factors of production" in the production process:

• Compensation of employees: payments for the use of labour by way of wages and salaries, overtime and bonus payments, etc.

• Gross (of depreciation) operating surplus: the surplus from production before taking account of interest, dividend and rental flows (which tend not to be directly related to the production of goods and services). This in effect represents the return to the owners of entrepreneurial, physical and financial capital employed by the business, plus depreciation.

• Other indirect taxes: taxes other than those levied on goods and services (described above). Other indirect taxes cover general business-related taxes, and include rates.

Collectively, these payments comprise value-added, and are alternatively known as factor incomes (for reasons discussed below).

5.4.3 Factors

As noted above the "Factors" row shows the use of the factors of production by each industry. The "Factors" column shows how the payments to those factors are distributed to the owners of those factors i.e. households, and in the case of retained earnings, the producer enterprises in which that surplus is generated; the distribution of factor incomes is discussed in the next section.

5.4.4 Institutional sectors

The "Producer enterprises", "Households" and "Government" rows and columns show income transfers between institutional sectors and income-related flows, such as tax payments. These payments include:

- Payments by households of:

– transfers to other (New Zealand) households: gifts, charity payments, etc;

– transfers overseas, largely consisting of gifts;

– pension fund contributions and non-life insurance premiums to financial intermediaries (insurance companies, etc);

– personal income tax to government.

- Payments by financial intermediaries of:

– pension fund benefits and non-life insurance claims to households;

– non-life insurance claims to producer enterprises;

– corporate income tax and non-life insurance claims to government.

- Payments by producer enterprises of:

– corporate income tax to government;

– non-life insurance premiums to financial intermediaries.

- Payments by government of:

– social benefits to households: unemployment, sickness and domestic purposes benefits and other forms of government provided income support;

– non-life insurance premiums to financial intermediaries;

– overseas transfers.

- Payments by the rest of the world of:

– transfers to households;

– income tax to the New Zealand government.

5.4.5 Investment

As noted above, the intersection of the "Investment" column with the "Commodities" row shows the purchase of fixed (physical) assets.

Finally, the values in the "Investment"/"Rest of the World" intersections represent the extent to which New Zealand must borrow from the rest of the world to finance capital expenditure.

6. The microeconomic foundations of general equilibrium

This section outlines the microeconomic foundations of CGE modelling. In particular, it uses the notions of consumer and producer maximisation to develop the model framework[6]

The basis for a CGE model is the circular flow of commodities in an economy. The principal agents in a stylised economy are households and firms. Households own the factors of production, which they rent to firms in return for some income stream. Firms employ these factors in order to produce goods and services for household consumption, which are financed via households' factor returns. A slightly more complicated framework may also include a government agent, but in many settings the government plays a passive role, simply redistributing incomes via tax collection and transfer payments.

Economic equilibrium is characterised by three conditions. First, firms' revenue equals the sum of the value of the inputs to production i.e. the costs of intermediate inputs and primary factors. This is the *zero profit* condition. Under the standard CGE model configuration, all markets are assumed to be perfectly competitive, so that new entrants will join the market until any positive profits are reduced to zero. Similarly, negative profits will force firms to leave the market until profit levels return to zero.

The second condition is that the aggregate supply of a commodity is exhausted by the demand for that commodity. Similarly, the supply of the primary factors is exhausted by the demand for those factors. This is the *market clearance* condition.

Finally, households' factors are fully employed, with none left idle. In return they receive payments which constitute household income, which in turn is exhausted by purchases of goods and services. This is the *income balance* condition. This is akin to the zero profit condition of firms: households fully employ their available factors and exhaust their income by purchasing goods, in the production of a utility good, which has a value equal to the sum of value of the purchased goods and a price equal to the marginal utility of income.

These three conditions, or identities, are the cornerstone of Walrasian equilibrium. They are used by CGE models to simultaneously solve for the set of prices and the allocation of goods and factors that characterise general equilibrium. Walrasian equilibrium is defined by the allocation at equilibrium, rather than by the process of exchange implied by reallocation. The means by which goods and factors are transferred between agents is not

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6 This section draws from Sue Wing (2003).

a feature of CGE models, and hence there is no requirement for money as a means of exchange. Instead each good is valued relative to the value of a selected good – the *numeraire good* – which is assigned a price of 1. Thus all prices in CGE models are in terms of prices relative to the numeraire. MORE ABOUT THIS LATER??

6.1 The algebra of the SAM

Consider our model economy, comprising N industries, each which produces a single commodity, and an unspecified number of households which jointly own an endowment of F different types of primary factors. The economy is closed to international flows. Three simplifying assumptions about the economy are made. First, there are no price distortions in the form of taxes or subsidies, nor any quantitative restrictions on the sale or purchase of goods. Second, households act collectively as a single representative agent who rents the primary factors in return for income. This income is in turn used to purchase the N commodities for the purpose of satisfying D types of demands (e.g. consumption and investment). Finally, each industry acts as a single representative firm that hires the primary factors (from households) and uses some quantities of the N commodities in the form of intermediate inputs so that it produces a quantity y of its own output.

For algebraic purposes, let the sets of factors, industries, commodities and final demands be represented by the following indices:

$i = \{1,\ldots,N\}$: the set of commodities

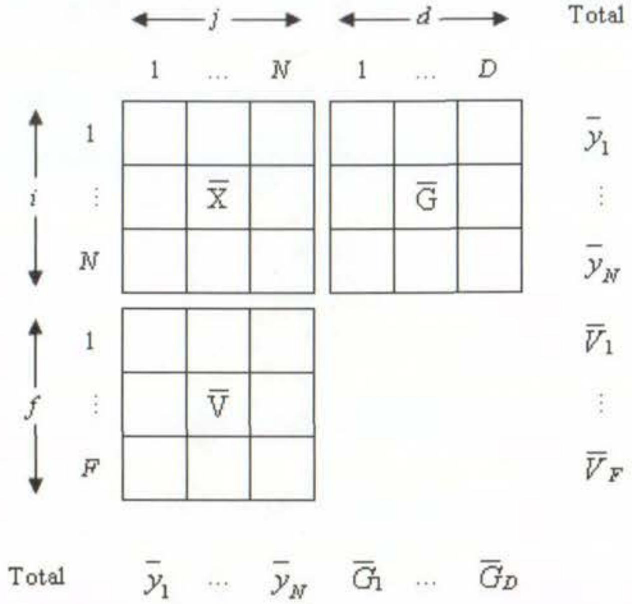
$j = \{1,\ldots,N\}$: the set of industry sectors

$f = \{1,\ldots,F\}$: the set of primary factors

$d = \{1,\ldots,D\}$: the set of final demands.

Thus the circular flow in this economy can be completely characterised by three matrices, which together form the stylised SAM illustrated in Figure 4. The three data matrices are: an N x N input-output matrix of industries' use of commodities as intermediate inputs, denoted by $\overline{X}$; an F x N matrix of primary factor inputs to industries, denoted by $\overline{V}$; and an N x D matrix of households use of commodities as final demands, denoted by $\overline{G}$.

Figure 4 A simple SAM



The three conditions that define balance in our SAM in Figure 4 can be established as follows. First, the zero profit condition requires that the gross output of the j^{th} sector, $\overline{y}_j$, equals the sum of the values of inputs of the i intermediate goods $\overline{x}_{ij}$ and the f primary factors $\overline{v}_{fj}$ that the industry employs in its production:

(1) $$\overline{y}_j = \sum_{i=1}^{N} \overline{x}_{ij} + \sum_{f=1}^{F} \overline{v}_{fj}$$

Second, commodity market balance requires that the aggregate value of supply of the i^{th} commodity, $\overline{y}_i$, must be equal to the aggregate value of demand for that commodity i.e. the sum of the values of the j intermediate uses of the good, $\overline{x}_{ij}$, and the d final demands for the good, $\overline{g}_{id}$. Thus:

(2) $$\overline{y}_i = \sum_{j=1}^{N} \overline{x}_{ij} + \sum_{d=1}^{D} \overline{g}_{id}$$

Similarly, factor market balance requires that the representative household's supply of each factor, $\overline{V}_f$, is fully employed by firms:

(3) $$\overline{V}_f = \sum_{j=1}^{N} \overline{v}_{fj}$$

Finally, the representative household's income, $\bar{m}$, is derived from the rental of its primary factors and is used to finance purchases of commodities to satisfy final demands. Thus, household income is equal to the sum of the elements of $\bar{V}$ which in turn is equal to the sum of the elements of $\bar{G}$:

(4) $$\bar{m} = \sum_{f=1}^{F} \bar{V}_f = \sum_{i=1}^{N} \sum_{d=1}^{D} \bar{g}_{id}$$

Equations (1) to (4) above formalise the accounting structure of the SAM shown in Figure 4. We now turn to formalising the behaviour of the household agent and firms – the so-called consumer's and producer's problems.

6.2 The consumer's problem

In our model economy, households, represented as a single agent, maximise their utility by purchasing quantities c_j of the N commodities subject to their income m and commodity prices p_i. Households may also consume goods and services for the purpose of savings, s, which are assumed to be exogenous and constant. Thus, the consumer's problem is defined as:

(5) $$\max_{c_i} U(c_1, ..., c_N) \qquad \text{subject to}$$

$$m = \sum_{i=1}^{N} p_i (c_i + s_i)$$

We also assume that the representative agent has Cobb-Douglas (C-D) preferences, so that her utility function is:

$$U = A_c c_1^{\alpha_1} c_2^{\alpha_2} ... c_N^{\alpha_N} = A_c \prod_{i=1}^{N} c_i^{\alpha_i}$$

with $\alpha_i + ... + \alpha_N = 1$. Equation (5) can be reformulated to reflect the household's role as a 'profit'-maximising producer of the utility good U. Output of U is generated by consumption, and its price p_u is the marginal utility of aggregate consumption. Thus, equation (5) is equivalent to the problem:

(6) $$\max_{c_i} pU - \sum_{i=1}^{N} p_i c_i \qquad \text{subject to}$$

$$U = A_c \prod_{i=1}^{N} c_i^{\alpha_i}$$

Solving this problem yields the representative agent's demand function for the consumption of the i^{th} commodity:

$$(7) \qquad c_i = \alpha_i \frac{\left(m - \sum_{i=1}^{N} p_i s_i \right)}{p_i}$$

Rearranging (7) gives:

$$\alpha_i = \frac{c_i p_i}{\left(m - \sum_{i=1}^{N} p_i s_i \right)}$$

where the denominator represents the total value of consumption. Thus, this rearranged expression shows that the exponents of the representative agent's utility function can be interpreted as each commodity's share of total consumption.

6.3 The producer's problem

Each producer maximises profit π by choosing levels of N intermediate inputs x and F primary factors v to produce output y, subject to the constraint of its production technology ϕ. The j^{th} producer's problem is thus:

$$(8) \qquad \max_{x_{ij}, v_{fj}} \pi_i = p_j y_j - \sum_{i=1}^{N} p_i x_{ij} - \sum_{f=1}^{F} w_f v_{fj} \qquad \text{subject to}$$

$$y_j = \phi_j \left(x_{1j}, \ldots, x_{Nj}; v_{1j}, \ldots, v_{Fj} \right)$$

As for households, we assume that producers in our model economy have Cobb-Douglas technology, so that the production function combining intermediate inputs and primary factors takes the form:

$$y_j = A_j \left(x_1^{\beta_1} x_2^{\beta_2} \ldots x_N^{\beta_N} \right) \left(v_1^{\gamma_1} v_2^{\gamma_2} \ldots v_F^{\gamma_F} \right) = A_j \prod_{i=1}^{N} x_{ij}^{\beta_{ij}} \prod_{f=1}^{F} v_{fj}^{\gamma_{fj}}$$

where $\beta_{ij} + \ldots + \beta_{Nj} + \gamma_{1j} + \ldots + \gamma_{Nj} = 1$. Solving the problem in (8) yields producer demands for intermediate inputs of commodities:

$$(9) \qquad x_{ij} = \beta_{ij} \frac{p_j y_j}{p_i}$$

and demands for primary factors:

(10) $$v_{fj} = \gamma_{fj} \frac{p_j y_j}{w_f}$$

These can be rearranged (again, in a similar manner as for households) to give

$$\beta_{ij} = \frac{p_i x_{ij}}{p_j y_j} \text{ and}$$

$$\gamma_{fj} = \frac{w_f v_{fj}}{p_j y_j}$$

respectively, showing that, as for consumption, the exponents of the Cobb-Douglas production function represent the shares of each input to the total value of inputs used in production.

6.4 General equilibrium

The three demand functions describing consumer's demand for commodities for consumption (equation (7)) and producer's demands for commodities as intermediate inputs and primary factors (equations (9) and (10)) form the core of the CGE model. These demand functions are tied together by the equilibrium conditions outlined in section 6.1, reformulated in terms of our model Cobb-Douglas economy.

The conditions for equilibrium in our Cobb-Douglas economy are as follows. As before, the market clearance condition implies that the quantity of each commodity produced must equal the sum of the quantities of commodities used as intermediate inputs by the *j* producers in the economy and as inputs to consumption and savings by the representative household. Thus, equation (1) can be specified as:

(11) $$y_i = \sum_{j=1}^{N} x_{ij} + c_i + s_i$$

Similarly, the quantities of primary factor *f* used by all producers must equal the representative agent's endowment of that factor, V_f. From equation (2) we have:

(12) $$V_f = \sum_{j=1}^{N} v_{fj}$$

The zero profit condition implies that the value of the output generated by producer *j* must equal the values of the inputs of the *i* intermediate goods and the *f* primary factors employed in production. Recall the profit maximisation problem of the producer in equation (8). Setting the left hand side of this problem to zero and rearranging yields:

(13) $$p_j y_j = \sum_{i=1}^{N} p_i x_{ij} - \sum_{f=1}^{F} w_f v_{fj}$$

which is the analogue of equation (3).

The income balance condition from equation (4), reformulated in terms of our Cobb-Douglas economy, becomes:

(14) $$m = \sum_{f=1}^{F} w_f V_f$$

Now that the equilibrium conditions have specified in terms of the Cobb-Douglas economy, the demand functions established earlier can be substituted into them to derive the actual equations used in our CGE model.

For the sake of simplicity, we assume that the endowment of the representative agent is fixed at the instant at which general equilibrium prevails (that is, the representative agent's stock of factors is fixed at this point in time). We can then substitute the consumer and producer demand functions for commodities (equations (7) and (9)) into the market clearance condition for commodities (equation (11)) to yield an excess demand function that defines the gap Δ^C between supply and demand in the market for N commodities. Thus, there are N excess demand equations for the commodity market:

(15) $$\Delta_i^C = \sum_{j=1}^{N} \beta_{ij} p_j y_j + \alpha_i \left(\sum_{f=1}^{F} w_f V_f - \sum_{j=1}^{N} p_j s_j \right) + p_i s_i - p_i y_i$$

Similarly, from the producers' demand functions for primary factors (equation (10) and the primary factor market clearance condition (equation (12)) we can derive the excess demand function that defines the difference Δ^F between primary factor demand and supply in the market for the F primary factors. Thus, there are F excess demand equations for the factor market:

(16) $$\Delta_f^F = \sum_{j=1}^{N} \gamma_{fj} \frac{p_j y_j}{w_f} - V_f$$

The zero profit condition requires that the absolute value of producers' profits is minimised to zero in general equilibrium. Thus, substituting equations (9) and (10) into the production function yields the N excess-demand functions that specify the per-unit excess profit (i.e. the excess of price over unit cost) Δ^{π} in each industry sector:

(17) $$\Delta_j^{\pi} = p_j - A_j \prod_{i=1}^{N} \left(\frac{p_i}{\beta_{ij}} \right)^{\beta_{ij}} \prod_{f=1}^{F} \left(\frac{w_f}{\gamma_{fj}} \right)^{\gamma_{fj}}$$

Finally, the income balance condition (equation (14)) can be re-written in terms of the excess of income over returns to the representative agent's endowment of primary factors, Δ^m:

(18) $$\Delta^m = \sum_{f=1}^{F} w_f V_f - m$$

The absolute value of each of the divergence factors in equations (15) to (18) – that is, $\Delta^C, \Delta^F, \Delta^{\pi}$ and Δ^m – is jointly minimised in general equilibrium. In other words, excesses in demand (or supply) in the commodity or factor markets, and producer or consumer income imbalances, are minimised to zero via the solution of an optimisation problem. The four equations which define excess demands and income imbalances (i.e. equations (15) to (18)) constitute the workings of the CGE model. We turn to the process by which this optimisation problem is solved next.

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6.4.1 Model formulation

The model is formulated such that it solves the minimisation of each of the equations (15) to (18) by determining the activity and price levels that support general equilibrium. Thus, we have a system of *2N+F* equations in *2N+F* unknowns: an *N*-vector of industry output (i.e. "activity") levels $\mathbf{y} = [y_1, ..., y_N]$, an *N*-vector of commodity prices $\mathbf{p} = [p_1, ..., p_N]$, an *F*-vector of factor prices $\mathbf{w} = [w_1, ..., w_F]$ and a scalar income level *m*. The general equilibrium solution is obtained by finding those activity levels and prices that solve the problem

(19) $$\xi(\mathbf{z}) = \mathbf{0}$$

in which $\mathbf{z} = [\mathbf{p}, \mathbf{w}, \mathbf{y}, m]'$ is the vector of stacked prices, activity levels and level of income and $\xi(.) = [\Delta^C, \Delta^F, \Delta^{\pi}, \Delta^m]'$ is the system of stacked excess demand functions. Equation (19) is referred to as the expression of general equilibrium in complementarity format, reflecting the complementary relationship between prices and excess demands, and between activity levels and profits.

To ensure that the general equilibrium of equation (19) is economically meaningful, prices, activity levels and the level of income are assumed to be positive and finite (i.e. $\mathbf{0} \le \mathbf{z} < \infty$). In the limit, as $\mathbf{z}$ approaches zero, equations (15), (17) and (18) all approach zero, and equation (16) tends to $-V_f$, implying that $\xi(\mathbf{0}) = [\mathbf{0}, \mathbf{-V}, \mathbf{0}, 0]' \le \mathbf{0}$. If $\mathbf{z}^*$ is a vector of prices and activity and income levels that support general equilibrium, it must be the case that $\mathbf{0} \le \mathbf{z}^*$ and $\xi(\mathbf{z}^*) = \mathbf{0}$. Thus, the problem in equation (19) may be compactly respecified as one of finding

(20) $\mathbf{z} \ge \mathbf{0}$ subject to

$$\xi(\mathbf{z}) \ge \mathbf{0}, \; \mathbf{z}'\xi(\mathbf{z}) = \mathbf{0}$$

which is a mathematical statement of Walras' Law that the sum of the values of market demands is equal to the value of market supplies.

6.4.2 Numerical calibration of the model using the SAM

Although Cobb-Douglas preferences and technology characterise an economy in relatively simple terms, equation (20) is still highly non-linear, and as a result there is no closed-form analytical solution for $\mathbf{z}$.[7] Hence the reason for the "C" in CGE models: general equilibrium systems with realistic utility and production functions must be calibrated on a SAM such as that illustrated in Figure 4. Calibration to a SAM generates a numerical optimization problem which can then be solved using optimization techniques.

Numerical calibration involves "fitting" the CGE model to the benchmark equilibrium recorded in the SAM. Calibration in our Cobb-Douglas economy is achieved by observing the symmetry between the equations specifying the accounting identities of the SAM (in equations (1) to (4)) and those defining general equilibrium (in equations (11) to (14)). In particular, we can see that the SAM identity equations are equivalent to their general equilibrium analogues provided that $\pi_j = 0$, $p_i x_{ij} = \overline{x}_{ij}$ and $w_f v_{fj} = \overline{v}_{fj}$. The first of these conditions holds via our assumption of zero profit. The second and third conditions hold provided that the prices of commodity *i* and factor *f* are equal to one. Thus calibration is achieved by treating each price in the model as an index with a value of unity in the benchmark, and by treating all values in the SAM as benchmark quantities. These assumptions allow the technical coefficients and elasticity parameters of the utility and production functions to be solved for directly:

(21) $$\alpha_i = \frac{\overline{g}_{iC}}{\overline{G}_C}$$

[7] While subject to a number of interpretations, "closed-form" in this instance refers to solution that can be expressed analytically in terms of a bounded number of well-known operations, and thus does not involve the use of, for example, root-finding algorithms.

(22) $$A_C = \bar{G}_C \Big/ \left(\prod_{i=1}^{N} \bar{g}_{iC}^{\alpha_i} \right)$$

(23) $$\beta_{ij} = \frac{\bar{x}_{ij}}{\bar{y}_j}$$

(24) $$\gamma_{fj} = \frac{\bar{v}_{fj}}{\bar{y}_j}$$

(25) $$A_j = \bar{y}_j \Big/ \left(\prod_{i=1}^{N} \bar{x}_{ij}^{\beta_{ij}} \prod_{f=1}^{F} \bar{v}_{fj}^{\gamma_{fj}} \right)$$

(26) $$s_i = \bar{g}_{iS}$$

(27) $$V_f = \bar{V}_f$$

(28) $$\bar{m} = \sum_{f=1}^{F} \bar{V}_f$$

Having specified these values for the model's coefficients, solving the problem in equation (20) will then set the quantities of the variables in the Cobb-Douglas economy equal to the values of the corresponding flows of the SAM (i.e. $x_{ij} = \bar{x}_{ij}$, $v_{fj} = \bar{v}_{fj}$ and $c_i = \bar{g}_{iC}$, replicating the benchmark equilibrium.

6.4.3 The solution of the CGE model in complementarity format

The calibration procedure outlined above transforms equation (20) into a square system of equations known as a *nonlinear complementarity problem* or NCP (Ferris and Pang, 1997). GAMS is one of a number of software packages that embodies optimisation algorithms required to solve this type of problem. The basic approach is similar to a Newton-type steepest-descent optimisation algorithm, which iteratively solves a sequence of linear complementarity problems (LCPs), each of which is a first-order Taylor series expansion of the non-linear function ξ. The LCP solved at each iteration thus involves finding

(29) $\mathbf{z} \geq \mathbf{0}$ subject to

$$\mathbf{q} + \mathbf{Mz} \geq \mathbf{0},\ \mathbf{z}'(\mathbf{q} + \mathbf{Mz}) = \mathbf{0}$$

where, linearizing ξ around $\mathbf{z}_{(k)}$, the state vector of prices, activity levels and income at iteration *k*, $\mathbf{q}(\mathbf{z}_{(k)}) = \nabla\xi(\mathbf{z}_{(k)}) - \xi(\mathbf{z}_{(k)})$ and $\mathbf{M}(\mathbf{z}_{(k)}) = \nabla\xi(\mathbf{z}_{(k)})$. Then, starting from an initial point $\mathbf{z}_{(0)}$, the solution of the problem in equation (21) at the k^{th} iteration $\mathbf{z}^*_{(k)}$ updates the value if $\mathbf{z}$ according to:

(30) $$\mathbf{z}_{(k+1)} = \mu_{(k)}\mathbf{z}^*_{(k)} + \left(1 - \mu_{(k)}\right)\mathbf{z}_{(k)}$$

where the parameter $\mu_{(k)}$ controls the length of the forward step that the model takes at each iteration. The convergence criterion for the algorithm consisting of equations (29) and (30) is just the numerical analogue of equation (19): $\left\|\xi\left(\mathbf{z}_{(k)}\right)\right\| < \varpi$, in which the scalar parameter ϖ is the maximum tolerance level of excess demands, profits or income at which the algorithm is deemed by the analyst to have converged to an equilibrium.

7. A simple CGE model

This section presents a simple, working CGE model, and builds on the theoretical principals outlined above.[8] The model presented has been kept deliberately simple for two reasons: first, it aids understanding on the part of the reader, and second, it can be run using the demo (i.e. free) version of GAMS.[9]

The model incorporates an equally simple counterfactual which crudely replicates the potential impact of a natural hazard event; specifically, the counterfactual involves an exogenous decrease in the supply of capital, mimicking the property loss of a hazard event.

7.1 Model notation

Sets (indices)

a	activities: agriculture (AGR), manufacturing (MFG), services (SRV)
c	commodities: agriculture (AGR-C), manufacturing (MCG-C), services (SRV-C)
f	factors: labour (LAB), capital (CAP)
h	households

Model parameters

ad_a	efficiency parameter in the production function for activity a
cpi	consumer price index (CPI)
$cwts_c$	weight of commodity c in the CPI
ica_{ca}	quantity of commodity c as intermediate input per unit of activity a
$shry_{hf}$	share of household h in the income of factor y
qfs_f	supply of factor f
α_{fa}	share of value-added for factor f in activity a
β_{ch}	share of household h consumption spending of commodity c
θ_{ac}	yield of output c per unit of activity a

[8] The model presented in this section draws from Lofgren (2003).

[9] As noted in section 3, the unlicensed version of GAMS can be obtained at http://www.gams.com/download .

Model variables

P_c	market price of commodity c
PA_a	price of activity a
PVA_a	value-added price of activity a
Q_c	output level in commodity c
QA_a	level of activity a
QF_{fa}	demand of factor f by activity a
QH_{ch}	consumption of commodity c by household h
$QINT_{ca}$	quantity of commodity c as intermediate input in activity a
YF_{hf}	income of household h from factor f
WF_f	price of factor f
YH_h	income of household h

Model equations

Production and commodity block

Activity production function:

$$QA_a = ad_a \prod_f QF_{fa}^{\alpha_{fa}}$$

Factor demand:

$$WF_f = \frac{\alpha_{fa} PVA_a QA_a}{QF_{fa}}$$

Intermediate input demand:

$$QINT_{ca} = ica_{ca} QA_a$$

Activity price:

$$PA_a = \sum_c \theta_{ac} P_c$$

Value-added price:

$$PVA_a = PA_a - \sum_c P_c ica_{ca}$$

Commodity output:

$$Q_c = \sum_a \theta_{ac} QA_a$$

Institution block

Factor income:

$$YF_{hf} = shry_{hf} WF_f \sum_a QF_{fa}$$

Household income:

$$YH_h = \sum_f YF_{hf}$$

Household demand:

$$QH_{ch} = \frac{\beta_{ch} YH_h}{P_c}$$

System constraint block

Factor market equilibrium:

$$\sum_a QF_{fa} = qfs_f$$

Output market equilibrium:

$$Q_c = \sum_h QH_{ch} + \sum_a QINT_{ca}$$

Price normalisation:

$$\sum_c cwts_c P_c = cpi$$

7.2 Model SAM

The social accounting matrix used in this model is presented in Table 1.

Table 1 EQC model SAM

	AGR	MFG	SRV	AGR-C	MFG-C	SRV-C	LAB	CAP	HHD	Total
AGR				245						245
MFG					280					280
SRV						235				235
AGR-C	60	40	20						125	245
MFG-C	40	60	30						150	280
SRV-C	20	30	35						150	235
LAB	62	55	85							202
CAP	63	95	65							223
HHD							202	223		425
Total	245	280	235	245	280	235	202	223	425	

Source: Adapted after Lofgren (2003)

7.3 GAMS code

This section presents the GAMS code for the model outlined above. Note that the code is self-contained; that is, the GAMS code listed includes all required, including the SAM in Table 1, for successful running of the model.

```
$title
$title MODEL: EQC Basic
$title Description: A simple working application of a CGE model,
$title    including a (simple) simulation of earthquake damage.
$title
$title Model based on Lofgren (2003)
$title
$title Mark Walton, July 2005
$title

* DEFINE UNDERLYING SETS (INDICES) ===============================
*

SETS

ac SAM accounts and other items
      /AGR agricultural activity
       MFG manufacturing activity
       SRV services activity
       AGR-C agricultural commodity
       MFG-C manufacturing commodity
       SRV-C services commodity
       LAB labor
       CAP capital
       HHD households
       TOTAL total account in SAM /

acnt(ac) all elements in AC except total
a(ac) activities
      /AGR, MFG, SRV/

c(ac) commodities
      /AGR-C, MFG-C, SRV-C/

f(ac) factors
      /LAB, CAP/

h(ac) households
      /HHD/
;
```

```
ALIAS (ac,acp), (c,cp), (f,fp) ;
acnt(ac) = YES; acnt('TOTAL') = NO; alias(acnt,acntp);


* SOCIAL ACCOUNTING MATRIX ========================================

* Read in benchmark social accounting matrix

TABLE SAM(AC,ACP) social accounting matrix

          AGR  MFG  SRV  AGR-CMFG-CSRV-CLAB  CAP  HHD
AGR                      245
MFG                           280
SRV                                235
AGR-C     60   40   20                            125
MFG-C     40   60   30                            150
SRV-C     20   30   35                            150
LAB       62   55   85
CAP       63   95   65
HHD                                     202    223
;


* Perform basic check that SAM is in balance:

PARAMETER
     tdiff(ac)  column minus row total for account ac ;

sam('total',acntp) = sum(acnt, sam(acnt,acntp));
sam(acnt,'total') = sum(acntp, sam(acnt,acntp));
tdiff(acnt) = sam('total',acnt)-sam(acnt,'total');

DISPLAY sam, tdiff;


* DEFINE MODEL PARAMETERS ========================================

PARAMETERS

     ad(a)            efficiency parameter in the production fn for a
     alpha(f,a)       share of value-added to factor f in activity a
     beta(c,h)        share of household consumption spending on
                      commodity c
     cpi              consumer price index
     cwts(c)          weight of commodity c in the cpi
```

```
     ica(c,a)         qnty of c as intermediate input per unit of
                      activity a
     qfs(f)           supply of factor f
     shry(h,f)        share for household h in the income of factor f
     theta(a,c)       yield of output c per unit of activity a
;


* ASSIGN BENCHMARK (INITIAL) VALUES FOR PARAMETERS AND VARIABLES

PARAMETERS

     p0(c)            benchmark price of commodity c
     pa0(a)           benchmark price of activity a
     pva0(a)          benchmark price of value-added for activity a
     q0(c)            benchmark output level of commodity c
     qa0(a)           benchmark output level of activity a
     qf0(f,a)         benchmark demand for factor f by activity a
     qint0(c,a)       benchmark demand for intermedaite good c by
                      activity a
     qh0(c,h)         benchmark consumption of commodity c by
                      household h
     wf0(f)           benchmark price of factor f
     yf0(h,f)         benchmark income of household h from factor f
     yh0(h)           benchmark income of household (total)
;


* PRODUCTION AND COMMODITY BLOCK ++++++++

* Benchmark prices set equal to 1:
p0(c) = 1;
pa0(a) = 1;
wf0(f) = 1;

* Benchmark production values derived from SAM:
pva0(a) = sum(f, sam(f,a)) / (sam(a,'total')/pa0(a));
q0(c) = sam('total',c)/p0(c);
qa0(a) = sam('total',a)/pa0(a);
qf0(f,a) = sam(f,a)/wf0(f);
qint0(c,a) = sam(c,a)/p0(c);

* Production parameters derived from SAM:
alpha(f,a) = sam(f,a) / sum(fp, sam(fp,a));
ad(a) = qa0(a) / prod(f, qf0(f,a)**alpha(f,a));
ica(c,a) = (sam(c,a)/p0(c)) / qa0(a);
```

```
theta(a,c) = (sam(a,c)/p0(c)) / qa0(a);


* INSTITUTION BLOCK ++++++++++++++++++++

* Benchmark consumption values derived from SAM:
qh0(c,h) = sam(c,h)/p0(c);
yf0(h,f) = sam(h,f);
yh0(h) = sam('total',h);

* Consumption parameters derived from SAM:
beta(c,h) = sam(c,h)/sum(cp, sam(cp,h));
shry(h,f) = sam(h,f)/sam('total',f);


* SYSTEM CONSTRAINT BLOCK ++++++++++++++

* Constraint parameters derived from SAM:
cwts(C) = SUM(H, SAM(C,H)) / SUM((CP,H), SAM(CP,H));
cpi = SUM(C, cwts(C)*P0(C));
qfs(F) = SAM(F,'TOTAL')/WF0(F);


* MODEL VARIABLES ==================================================

VARIABLES

     P(C)            price of commodity c
     PA(A)           price of activity a
     PVA(A)          value-added (or net) price for activity a
     Q(C)            output level for commodity c
     QA(A)           level of activity a
     QF(F,A)         quantity demanded of factor f from activity a
     QH(C,H)         quantity consumed of commodity c by household h
     QINT(C,A)       qnty of commodity c as intermediate input to
                     activity a
     WF(F)           price of factor f
     YF(H,F)         income of household h from factor f
     YH(H)      income of household h
;


* INITIALIZING ALL VARIABLES +++++++++++

* Initial value of variables set to SAM values:
P.L(C) = p0(c) ;
```

```
PA.L(A) = pa0(a) ;
PVA.L(A)= pva0(a) ;
Q.L(C) = q0(c) ;
QA.L(A) = qa0(a) ;
QF.L(F,A) = qf0(f,a) ;
QH.L(C,H) = qh0(c,h) ;
QINT.L(C,A) = qint0(c,a) ;
YF.L(H,F) = yf0(h,f) ;
WF.L(F) = wf0(f) ;
YH.L(H) = yh0(h) ;


*DISPLAY ++++++++++++++++++++++++++++++

DISPLAY
ad, alpha, beta, cpi, cwts, qfs, shry, theta,
## Initial prices and quantities, P.L, PA.L, PVA.L, Q.L, QA.L, QF.L,
QH.L, WF.L, YF.L, YH.L
;


* DEFINE MODEL EQUATIONS ==========================================

EQUATIONS

* PRODUCTION AND COMMODITY BLOCK ++++++++
     PRODFN(a)        Cobb-Douglas production function for activity a
     FACDEM(f,a)      demand for factor f from activity a
     INTDEM(c,a)      intermediate demand for commodity c from
                      activity a
     OUTPUTFN(c)      output of commodity c
     PADEF(a)         price for activity a
     PVADEF(a)        value-added price for activity a

* INSTITUTION BLOCK ++++++++++++++++++++
     FACTTRNS(h,f)    transfer of income from factor f to h-hold h
     HHDINC(h)        income of household h
     HHDEM(c,h)       consumption demand for household h & commodity
                      c

* SYSTEM CONSTRAINT BLOCK +++++++++++++++
     FACTEQ(f)        market equilibrium condition for factor f
     EQAGRC           market equilibrium condition for commodity AGR-
                      C
     EQMFGC           market equilibrium condition for commodity MFG-
                      C
```

```
     PNORM      price normalization
;


* PRODUCTION AND COMMODITY BLOCK ++++++++

PRODFN(a)..       QA(a) =e= ad(a)*prod(f, QF(f,a)**alpha(f,a));
FACDEM(f,a)..     WF(f) =e= alpha(f,a)*PVA(a)*QA(a) / QF(f,a);
INTDEM(c,a)..     QINT(c,a) =e= ica(c,a)*QA(a);

OUTPUTFN(c)..     Q(c) =e= sum(a, theta(a,c)*QA(a));

PADEF(a)..        PA(a) =e= sum(c, theta(a,c)*P(c));
PVADEF(a)..       PVA(a) =e= PA(a)-sum(c, P(c)*ica(c,a));


* INSTITUTION BLOCK ++++++++++++++++++++

FACTTRNS(h,f).. YF(h,f) =e= shry(h,f)*WF(f)*sum(a, QF(f,a));

HHDINC(h)..       YH(h) =e= sum(f, YF(h,f));

HHDEM(c,h)..      QH(c,h) =e= beta(c,h)*YH(h)/P(c);

* SYSTEM CONSTRAINT BLOCK ++++++++++++++

FACTEQ(f)..       sum(a, QF(f,a)) =e= qfs(f);
EQAGRC..          Q('AGR-C') =e= sum(h, QH('AGR-C',h)) +
                  sum(a, QINT('AGR-C',a));
EQMFGC..          Q('MFG-C') =E= sum(h, QH('MFG-C',h)) +
                  sum(a, QINT('MFG-C',a));

PNORM..           sum(c, cwts(c)*P(c)) =e= cpi;


* DEFINE THE MODEL ==============================================

MODEL
     EQC  Basic CGE model        /ALL/
;


* TEST THE MODEL ================================================
* Test the integrity of the model by solving with zero iterations
* If model correctly specified, solution will replicate the
* benchmark (i.e. initial data) without equation infeasibilities
```

```
DISPLAY "Benchmark replication test" ;
eqc.iterlim = 0 ;
SOLVE EQC using mcp ;


* ESTABLISH COUNTERFACTUAL =======================================
* Counterfactual simulates an exogenous decrease in the endowment of
* capital, replicating, in a simple fashion, the effect of property
* damage following an earthquake.

* SET AND PARAMETERS FOR REPORTS ++++++++

SET
     SIM simulations
     /BASE     base simulation
      CAPCHG        change in capital stock /
;

PARAMETERS

* Establish parameter to allow an exogenous change in the value of
* the capital stock (replicating the impact of a hazard):
     qfscapsim(sim)       capital   supply   for   simulation   sim
                          (experiment parameter)

* Establish parameters to collect simulation results:
     qfsrep(f,sim)        supply  of  factor  f  for  simulation  sim
                          (value used)
     prep(c,sim)          demander price for commodity c
     parep(a,sim)         price of activity a
     qrep(c,sim)          output level for commodity c
     qarep(a,sim)         level of activity a
     qfrep(f,a,sim)       demand for factor f from activity a
     qhrep(c,h,sim)       consumption of commodity c by household h
     wfrep(f,sim)         price of factor f

     yfrep(h,f,sim)       income of household h from factor f
     yhrep(h,sim)         income of household h
     samrep(sim,ac,acp)   sam computed from model solution
     balchk(ac,sim)       column minus row total for account ac in
                          sam
;


* Set base case value of capital stock equal to benchmark value:
```

```
qfscapsim('BASE') = qfs('CAP');

* For counterfactual, exogenously decrease supply of capital stock
by 30%:
qfscapsim('CAPCHG') = 0.7*qfs('CAP');

DISPLAY qfscapsim;


* RUN BASE AND COUNTERFACTUAL SIMULATIONS ++++

LOOP(sim,

 qfs('CAP') = qfscapsim(sim);

eqc.iterlim = 1000 ;
SOLVE EQC using mcp ;

* Collect simulation results:
qfsrep(f,sim) = qfs(f);

prep(c,sim) = p.l(c);
parep(a,sim) = pa.l(a);
qrep(c,sim) = q.l(c);
qarep(a,sim) = qa.l(a);
qfrep(f,a,sim) = qf.l(f,a);
qhrep(c,h,sim) = qh.l(c,h);
wfrep(f,sim) = wf.l(f);
yfrep(h,f,sim) = yf.l(h,f);
yhrep(h,sim) = yh.l(h);

* Payments from activities
samrep(sim,f,a) = WF.L(f)*QF.L(f,a);
* Payments from commodities
samrep(sim,a,c) = P.L(c)*theta(a,c)*QA.L(a);
* Payments from factors
samrep(sim,h,f) = YF.L(h,f);
* Payments from households
samrep(sim,c,h) = P.L(c)*QH.L(c,h);

);


* Computing totals for SAMREP
samrep(sim,'total',acntp) = sum(acnt, samrep(sim,acnt,acntp));
samrep(sim,acnt,'total') = sum(acntp, samrep(sim,acnt,acntp));
```

```
* Check that SAMREP is balanced
balchk(acnt,sim) = samrep(sim,'total',acnt)-
     samrep(sim,acnt,'total');


OPTION qfrep:3:1:1, qhrep:3:1:1, yfrep:3:1:1, samrep:3:1:1;

DISPLAY
qfsrep, prep, parep, qrep, qarep, qfrep, qhrep, wfrep, yfrep, yhrep,
samrep, balchk
;


* Establish parameters to reporting percentage change from BASE for
* model variables and for selected other data.
PARAMETERS
     qfsrepp(f,sim)         supply of factor f for simulation sim
                            (%ch)
     prepp(c,sim)           demander price for commodity c (%ch)
     parepp(a,sim)          price of activity a (%ch)
     qrepp(c,sim)           output level for commodity c (%ch)
     qarepp(a,sim)          level of activity a (%ch)
     qfrepp(f,a,sim)        demand for factor f from activity a (%ch)
     qhrepp(c,h,sim)        consumption of commodity c by household h
                            (%ch)
     yfrepp(h,f,sim)        income of household h from factor f (%ch)
     wfrepp(f,sim)          price of factor f (%ch)
     yhrepp(h,sim)          income of household h (%ch)
     samrepp(sim,ac,acp)    sam computed from model solution (%ch by
                            cell)
;

qfsrepp(f,sim) = 100*(qfsrep(f,sim)/qfsrep(f,'BASE')-1);
prepp(c,sim) = 100*(prep(c,sim)/prep(c,'BASE')-1);
parepp(a,sim) = 100*(parep(a,sim)/parep(a,'BASE')-1);
qrepp(c,sim) = 100*(qrep(c,sim)/qrep(c,'BASE')-1);
qarepp(a,sim) = 100*(qarep(a,sim)/qarep(a,'BASE')-1);
qfrepp(f,a,sim) = 100*(qfrep(f,a,sim)/qfrep(f,a,'BASE')-1);
qhrepp(c,h,sim) = 100*(qhrep(c,h,sim)/qhrep(c,h,'BASE')-1);
wfrepp(f,sim) = 100*(wfrep(f,sim)/wfrep(f,'BASE')-1);
yfrepp(h,f,sim) = 100*(yfrep(h,f,sim)/yfrep(h,f,'BASE')-1);
yhrepp(h,sim) = 100*(yhrep(h,sim)/yhrep(h,'BASE')-1);
samrepp(sim,ac,acp)$samrep('BASE',ac,acp)
 = 100*(samrep(sim,ac,acp)/samrep('BASE',ac,acp)-1);
```

```
OPTION qfrepp:3:1:1, qhrepp:3:1:1, yfrepp:3:1:1, samrepp:3:1:1;


DISPLAY
qfsrepp, prepp, parepp, qrepp, qarepp, qfrepp, qhrepp, wfrepp,
yfrepp, yhrepp, samrepp
;

$ontext
$offtext
```

8. Resources Page

Introduction

In this section we provide a central directory of all the resources that are referenced from throughout the various parts of this website. For the most part, these resources will consist of GAMS-readable data files and GAMS programs.

The files can be downloaded by right-clicking the links and selecting "Save Target As…", or they can be viewed within the browser simply by clicking on the links.

Apart from making sure that file path names correspond to the appropriate directories (especially important for input files but also desirable for output files), all of the GAMS programs listed here should compile and run without producing any errors. Be sure to read any notes located at the top of the GAMS programs and data files. All GAMS programs and data files are ascii (text) files.

Data files

GAMS-readable

1. NZ 1996 Input Output table (126).csv

This file is a GAMS-readable version of the 126-industry 1996 input-output table published by Statistics New Zealand. It has been saved in the Excel .csv format, the subtotals and totals have been removed, and the row and column headers have been replaced with the set elements listed in NZ 1996 Input Output table labels (126).set.

2. NZ 1996 Input Output table labels (126).set

This file contains the set elements that correspond to the row and column headings of the New Zealand 126-industry 1996 input-output table. While we have created the element names (i.e. they are not necessarily the official symbols), the description of each element is the official Statistics New Zealand description.

3. NZ 1996 Input Output (126 to 4).map

This file is a mapping scheme that associates sectors in the 126-industry table with sectors in our 4-sector illustrative input-output table, for the purpose of aggregating the 126-industry table. It can be used as a template to create a mapping scheme for aggregating the input-output table to any desired dimension.

4. NZ 1996 FTE Employment (126).csv

This GAMS-readable file contains full time equivalent employment numbers for each of the 126 industry sectors in the 1996 input-output table. The employment data comes from the 1996 New Zealand Census of Population and Dwellings, and represents the labour force status for the usually resident population aged 15 years and over. In computing FTEs, we assume that 1 full time is equal to 2 part time.

5. NZ 1996 150 Use industries.set

The industry set elements and descriptions that correspond to the column headings of the 1996 New Zealand Supply and Use tables. There are actually only 126 industries but some of them are distinguished by characterisations such as market and non-market. Hence there are 150 Supply and Use industry designations.

6. NZ 1996 210 Commodities.set

The commodity set elements and descriptions that correspond to the row headings of the 1996 New Zealand Supply and Use tables.

7. NZ 1996 126 Industries.set

The set elements and descriptions that correspond to the 126 industry sector headings of the 1996 New Zealand input-output and imports table. Note that this file contains only the 126 industry labels. For industries as well as final demand and value-added sectors, see NZ 1996 Input Output table labels (126).set.

8. NZ 1996 49 Industries.set

Set elements and descriptions for 49 industry sectors.

9. NZ 1996 64 Commodities.set

Set elements and descriptions for 64 commodities.

10. NZ 1996 Use mapping (150-49).map

Mapping scheme for aggregating the 150 Use industries into 49 industry sectors.

11. NZ 1996 Commodity mapping (210-64).map

Mapping scheme for aggregating 210 commodities into 64 commodities.

12. NZ 1996 Industry mapping (126-49).map

Mapping scheme for aggregating 126 industries into 49 industry sectors.

13. NZ 1996 Supply - 210 commodity by 150 Use industries.csv

A GAMS-readable version of the 1996 supply table as published by Statistics New Zealand. It has been saved in the Excel .csv format, the subtotals and totals have been removed, and the row and column headers have been replaced with the set elements listed in NZ 1996 150 Use industries.set and NZ 1996 210 Commodities.set.

14. NZ 1996 Use - 210 commodity by 150 Use industries.csv

A GAMS-readable version of the 1996 use table as published by Statistics New Zealand. It has been saved in the Excel .csv format, the subtotals and totals have been removed, and the row and column headers have been replaced with the set elements listed in NZ 1996 150 Use industries.set and NZ 1996 210 Commodities.set.

15. NZ 1996 Imports - 210 Commodity by Use.csv

A GAMS-readable version of the 1996 imports table as published by Statistics New Zealand. It has been saved in the Excel .csv format, the subtotals and totals have been removed, and the row and column headers have been replaced with the set elements listed in NZ 1996 126 Industries.set and NZ 1996 210 Commodities.set

16. XXX

ccc

GAMS Programs

1. Aggregate Input Output table.gms

A program to aggregate the New Zealand 126-industry 1996 input-output table [some additional explanatory notes].

2. Input Output multipliers.gms

This program will read in an input-output table and produce a series of economic impact multipliers.

3. Aggregate Supply and Use.gms

This program is not referenced elsewhere on this website. It includes some GAMS code that is probably beyond the capabilities of a beginner. This program can be used as a starting point for users contemplating building a Social Accounting Matrix (SAM) from the raw input-output files as supplied by Statistics New Zealand.

4. 2x2c.gms

A simple 2-good, 2-factor CGE model formulated using MPSGE in scalar syntax.

5. 2x2d.gms

Model 2x2c with some "Report" variables added.

Useful websites

The websites listed here represent just a small fraction of the CGE modelling resources that can be found on the web. For those wishing to pursue the topic, especially those planning to use GAMS as a means of formulating and solving CGE models, we believe these sites represent a useful place to start.

http://www.gams.com. The site of GAMS Development Corporation. Besides information about the GAMS software, this site is a useful gateway to many other resources to do with economic modelling. In particular, see the *Contributed Documentation* section.

http://debreu.colorado.edu/. Professor Tom Rutherford from the University of Colorado at Boulder is the author of the MPSGE software. His website contains a wealth of information on CGE modelling issues and applications.

http://www.ifpri.org/. The Trade and Macroeconomics Division of IFPRI publish many papers related to the construction of SAMs and the application of CGE models. From the IFPRI home page, go to *Research*, then *Research Divisions*, and then select *Trade and Macroeconomics*.

http://unstats.un.org/unsd/sna1993/toctop.asp. Chapter XX of the United Nation's *System of National Accounts 1993* provides detailed guidelines on the construction of Social Accounting Matrices.

http://www.gtap.agecon.purdue.edu/. The Global Trade and Analysis Project (GTAP) maintains a website providing access to a wide range of CGE modelling resources.

http://www.monash.edu.au/policy/. Researchers at the Centre of Policy Studies (CoPS) at Monash University have a long history of CGE modelling. Their website contains links to many valuable resources. You can also learn about GEMPACK, an alternative to GAMS for solving CGE models, from the CoPS website.

http://www.rri.wvu.edu/WebBook/Schreiner/contents.htm. This site contains an online book entitled *Computable general equilibrium modelling for regional analysis*.

http://www.wau.nl/wub/wep/nr9604/wep04.htm. This site contains a complete description (model structure and data) of WAGEM, the

Wageningen Applied General Equilibrium Model, which is used for agricultural and environmental policy analysis.

http://www.nottingham.ac.uk/~lezgr/teaching/CGE/agenotes.htm. Applied general equilibrium modelling course notes by Geoffrey Reed and Adam Blake.

Other documents

In this section we make available a collection of documents that we have "found" over the past year or so on the web, and which are not referenced from elsewhere on this website.

SAMs and CGE models

Social accounting matrices and applied general equilibrium models [Kehoe SAMs and CGEs.pdf] by Timothy Kehoe.

A primer on static applied general equilibrium models [Kehoe CGE primer.pdf] by Patrick Kehoe and Timothy Kehoe.

CGE Analyses of earthquakes

Estimating indirect economic losses from electricity lifeline disruption following a catastrophic earthquake in Memphis, TN [Guha - Lifelines and cge.pdf] by G. Guha.

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